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KIRK & BELFIELD

CHICAGO

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THE MODEL

ELEMENTARY

ARITHMETIC;

INCLUDING

ORAL AND WRITTEN EXERCISES.

BY

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Principals of Public Schools, Chicago,

AUTHORS OF MODEL ARITHMETIC, ETC.

CHICAGO:

GEO. SHERWOOD & CO.

PREFACE.

THIS book is designed both as an introduction to the Model Arithmetic, and as a text-book of sufficient comprehensiveness for those who do not complete the full grammar-school course. It has been the aim of the authors to present in it the accuracy of statement, clearness in discussion, and fullness of illustration, which are believed to characterize their larger work. The matter, aside from definitions, etc., is new, and has been tested in the school-room.

The introduction will be found of great use, not only in developing the idea of number, but in affording material for constant drill in fundamental operations.

Among the features which distinguish the book from many of its class, will be noticed the absence of pictures of common objects. No space is wasted in representing, by the engraver's art, that which every teacher presents to the eye of the pupil. Teachers every-where recognize the fact that the object itself is better than the picture of it; that, for instance, the actual division of an apple into halves, quarters, etc., in the presence of a class, produces a clearer and more lasting conception of the idea of a fraction, than is secured by any picture, however perfect.

The work, though elementary, is not designed as a plaything, but is an earnest attempt to assist in training the child's mind, and in fitting him for active life. As such, it is commended to the public.

K. & B.

Chicago, April, 1876.

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SUGGESTIONS TO TEACHERS.

The eight lessons that immediately follow have been arranged and introduced at the beginning of this book for the purpose of giving pupils a thorough drill in the use of the digits from 2 to 9 inclusive. Though for convenience of presentation the limit of the processes is confined to 12 times —, the teacher may extend the operations of addition and subtraction at will. The limit of 100, however, is all that is desirable with numbers less than 9.

The treatment of each number includes the four processes of addition, subtraction, multiplication and division, and recognizes the fact that they are intimately and naturally connected, and that one operation may be said to include all the others, for all the operations are only the comparison of numbers, one with another. They are, in fact, different methods of reading the same general relation of numbers, thus: $4+4=8$ is read 4 *and* 4 equals 8; $2\times4=8$ is read 2 *times* 4 equals 8; $4-2=2$ is read 4 *less* 2 *equals* 2; and $4\div2=2$ is read 2 is contained in 4, twice, or 2 can be taken twice from 4, or 2 is one-half of 4.

The consideration of these processes does not contemplate the use of the *terms add, subtract, multiply* or *divide*, as these terms will be more profitably discussed hereafter, but it is designed to make pupils familiar first with the processes themselves, and with the use of terms easily understood. It is assumed that pupils know how to count, and are familiar with the forms, names and values of the significant figures. While the teacher will be greatly assisted in this instruction by the use of lines or dots upon the board, or by a numeral frame, or small objects, such as grains of corn, beans, etc., it must not be forgotten that the memory should be made an important factor in the mastery of these relations. Constant appeals must be made to the pupil's power to gather and retain through frequent repetition.

Immediately succeeding the treatment of the abstract number, a variety of exercises, both abstract and denominate, is given as models for the teacher. These exercises may be variously diversified and extended, according to the skill of the teacher and the necessities of the pupils. The lessons should be used as follows: as for instance, Lesson I; (1) reads 2, 4, 6, 8, etc., 24; (2) 1, 3, 5, etc., 23; (3) 2, 4, 6, etc., 24; (4) 24, 22, etc., 2; (5) 23, 21, etc., 1; (6) once, twice, 3 times, etc., 12 times.

ADDITION TABLE.

1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9
2	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8
	3	3+1	3+2	3+3	3+4	3+5	3+6	3+7
2+9		4	4+1	4+2	4+3	4+4	4+5	4+6
3+8	3+9		5	5+1	5+2	5+3	5+4	5+5
4+7	4+8	4+9		6	6+1	6+2	6+3	6+4
5+6	5+7	5+8	5+9		7	7+1	7+2	7+3
6+5	6+6	6+7	6+8	6+9		8	8+1	8+2
7+4	7+5	7+6	7+7	7+8	7+9		9	9+1
8+3	8+4	8+5	8+6	8+7	8+8	8+9		10
9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	
11	12	13	14	15	16	17	18	

MULTIPLICATION TABLE.

	1's	2's	3's	4's	5's	6's	7's	8's	9's	10's	11's	12's	
1	1	2	3	4	5	6	7	8	9	10	11	12	*1*
2	2	4	6	8	10	12	14	16	18	20	22	24	*2*
3	3	6	9	12	15	18	21	24	27	30	33	36	*3*
4	4	8	12	16	20	24	28	32	36	40	44	48	*4*
5	5	10	15	20	25	30	35	40	45	50	55	60	*5*
6	6	12	18	24	30	36	42	48	54	60	66	72	*6*
7	7	14	21	28	35	42	49	56	63	70	77	84	*7*
8	8	16	24	32	40	48	56	64	72	80	88	96	*8*
9	9	18	27	36	45	54	63	72	81	90	99	108	*9*
10	10	20	30	40	50	60	70	80	90	100	110	120	*10*
11	11	22	33	44	55	66	77	88	99	110	121	132	*11*
12	12	24	36	48	60	72	84	96	108	120	132	144	*12*

PRACTICE TABLE.

A	B	C	D	E	F	G	H	I	J
1	9	12	8	4	7	5	3	6	2
3	5	1	4	12	6	12	7	12	9
5	3	3	1	8	12	8	9	1	3
7	1	9	7	1	9	3	12	10	7
9	7	8	3	11	1	7	4	8	4
11	2	5	9	9	11	1	8	4	10
2	11	7	12	7	8	4	11	2	6
4	8	10	5	5	4	11	1	5	12
6	4	6	11	3	10	6	5	3	1
8	12	4	6	2	5	2	10	7	11
10	6	8	10	10	2	10	6	9	8
12	10	11	2	6	3	9	2	11	5

The above table is to be used as follows:

1. ADDITION. Let the pupil add to each number in a given column a number announced by the teacher, the pupil stating the successive *sums* only. Thus, the teacher will say, "Column A, add 5;" the pupil selected answers *as rapidly as possible*, "6, 8, 10, 12, 14," etc.
2. SUBTRACTION. Let the pupil subtract from a given number each number in a given column. Thus, the teacher announces "Column B, subtract from 17." The pupil subtracts each number, mentally, and states the result only, "8, 12, 14, 16," etc.
3. MULTIPLICATION. The teacher selects a column and a multiplier. The pupil announces the product, using each number as a multiplicand. Thus, "C by 5," results in 60, 5, 15, 45, etc.
4. DIVISION. The teacher selects a column and announces a dividend; the pupil states the quotient, using each number as a divisor. It is not necessary that the dividend selected be a multiple of any of the divisors. Thus, the teacher may announce "D, dividend 20;" the pupil responds $2\frac{4}{8}$, 5, 20, $2\frac{6}{7}$, $6\frac{2}{3}$, etc.

In all of these exercises *accuracy* should be insisted upon from the beginning; rapidity should not be expected at first, but will result from constant practice.

MODEL

ELEMENTARY ARITHMETIC.

INTRODUCTION.

LESSON I.

DEVELOPMENT OF NUMBERS BY 2's.

1. 2+2+2+2+2+2+2+2+2+2+2+2=?

2. 1+2+2+2+2+2+2+2+2+2+2+2=?

3. 1×2=? 2×2=? 3×2=? 4×2=? 5×2=?
6×2=? 7×2=? 8×2=? 9×2=? 10×2=?
11×2=? 12×2=?

4. 24−2−2−2−2−2−2−2−2−2−2−2=?

5. 23−2−2−2−2−2−2−2−2−2−2−2=?

6. 2÷2=? 4÷2=? 6÷2=? 8÷2=? 10÷2=?
12÷2=? 14÷2=? 16÷2=? 18÷2=? 20÷2=?
22÷2=? 24÷2=?

---

7. 4 is 2 more than what number? 2 less than what number?

8. 15 is 2 more than what number? 2 less than what number?

9. 12 is twice what number? 22 is twice what number?

10. What number should be doubled to obtain 24? What number is contained in 18 twice?

11. What number is 2 more than 18? 2 less than 16?

12. What number should be added to 12 to obtain 14? 2 can be taken from 6 how many times?

13. 6 is one half what number? 8 is one half what number?

14. 11 is one half what number? 18 is twice what number?

15. 12 is double what number? Of what number is 7 one half?

16. 9 is one half what number? What five equal numbers equal 10?

17. 13 is equal to what six equal numbers and one more? 8 times 2 and 1 more equals what number?

18. 11 times 2 and 1 more equals what number?

---

19. A boy had 16 cents, and gave 2 cents to his sister. How many cents had he left?

20. Alfred had 22 marbles, and found 2 more. How many did he then have?

21. If one yard of cloth costs 6 dollars, how much will 2 yards cost?

22. John has 9 chestnuts, and his sister has twice as many. How many has his sister?

23. If one top costs 2 cents, how many tops may be bought for 16 cents?

24. How many two-cent stamps may be bought for 24 cents?

25. A girl paid 22 cents for a book, and one half as much for a slate. How much did she pay for the slate?

26. Paid 9 cents for a lead pencil, and twice as much and 1 cent more for some paper. How much was paid for the paper?

LESSON II.

DEVELOPMENT OF NUMBERS BY 3's.

1. 3+3+3+3+3+3+3+3+3+3+3+3=?

2. 2+3+3+3+3+3+3+3+3+3+3+3=?

3. 1+3+3+3+3+3+3+3+3+3+3+3=?

4. 1×3=? 2×3=? 3×3=? 4×3=? 5×3=? 6×3=? 7×3=? 8×3=? 9×3=? 10×3=? 11×3=? 12×3=?

5. 36−3−3−3−3−3−3−3−3−3−3−3=?

6. 35−3−3−3−3−3−3−3−3−3−3−3=?

7. 34−3−3−3−3−3−3−3−3−3−3−3=?

8. 3÷3=? 6÷3=? 9÷3=? 12÷3=? 15÷3=? 18÷3=? 21÷3=? 24÷3=? 27÷3=? 30÷3=? 33÷3=? 36÷3=?

9. 12 is 3 more than what number? 3 less than what number?

10. 26 is 3 more than what number? 3 less than what number?

11. 6 times 3 equals what number? 9 times 3 equals what number?

12. 15 is 3 times what number? 27 is 3 times what number?

13. What number is contained 3 times in 24? What number is equal to 3 times 7?

14. What number is 3 less than 22? 3 more than 28?

15. 36 is 3 times what number? 3 can be taken from 12 how many times?

16. Of what number is 8 one third ? 6 is one third of what number?

17. What 2 equal numbers are contained in 6? What 4 equal numbers equal 12?

18. 7 times 3 and 2 more equals what number? 9 times 3 and one more equals what number?

19. What is one of the 6 equal numbers contained in 18? 3 is contained in 27 how many times?

---

20. Henry had 25 dollars, and earned 3 dollars more. How many did he then have?

21. A girl having 31 cents, spent 3 cents for pins. How many cents did she have left?

22. One ton of coal costs 8 dollars. How much will 3 tons cost at the same rate?

23. James had 12 pigeons, and his father had 3 times as many. How many pigeons had his father?

24. One yard of ribbon costs 3 cents. At the same rate, how many yards may be bought for 18 cents?

25. How many yards of cloth, at 3 dollars a yard, may be bought for 36 dollars?

26. One third of 24 dollars was paid for a hat. How much did it cost?

27. A vest cost 9 dollars, and a coat 3 times as much, and 2 dollars more. How much did the coat cost?

LESSON III.

DEVELOPMENT OF NUMBERS BY 4's.

1. 4+4+4+4+4+4+4+4+4+4+4+4+4=?
2. 3+4+4+4+4+4+4+4+4+4+4+4+4=?
3. 2+4+4+4+4+4+4+4+4+4+4+4+4=?
4. 1+4+4+4+4+4+4+4+4+4+4+4+4=?
5. 1×4=? 2×4=? 3×4=? 4×4=? 5×4=? 6×4=? 7×4=? 8×4=? 9×4=? 10×4=? 11×4=? 12×4=?
6. 48−4−4−4−4−4−4−4−4−4−4−4−4=?
7. 47−4−4−4−4−4−4−4−4−4−4−4−4=?
8. 46−4−4−4−4−4−4−4−4−4−4−4−4=?
9. 45−4−4−4−4−4−4−4−4−4−4−4−4=?
10. 4÷4=? 8÷4=? 12÷4=? 16÷4=? 20÷4=? 24÷4=? 28÷4=? 32÷4=? 36÷4=? 40÷4=? 44÷4=? 48÷4=?

11. 20 is 4 more than what number? 4 less than what number?

12. 38 is 4 more than what number? 4 less than what number?

13. 28 is four times what number? 48 is 4 times what number?

14. What number is contained 4 times in 48? 4 times in 36?

15. What number is 4 more than 27? 4 less that 45?

16. What number should be added to 33 to obtain 37? 4 can be taken from 20 how many times?

17. 3 is one fourth what number? 8 is one fourth what number?

18. Of what number is 7 one-fourth? 6 is one fourth what number?

19. What number should be added to 4 times 7 to obtain 32? 4 times 6 and 3 more equals what number?

20. 9 times 4 and 2 more equals what number? 4 is contained in 48 how many times?

---

21. A man had 35 cents, and gave 4 of them for some apples. How many had he left?

22. Grace had 39 nuts, and found 4 more. How many had she then?

23. How much will 4 oranges cost, if 1 orange costs 9 cents?

24. James has 12 dollars, and his father has 4 times as many. How many has his father?

25. If one hat is worth 4 dollars, how many hats may be bought for 28 dollars?

26. Julia is one fourth as old as her mother, and her mother is 36 years old. How old is Julia?

27. A boy had 12 cents, and his sister had 4 times as many. How many cents had his sister?

28. A girl is 9 years old, and her father is 4 times as old, and 3 years more. How old is her father?

29. A person had 24 dollars, and put it into piles of 4 dollars each. How many piles were there?

30. Anna bought 8 pears at 4 cents each, and had 3 cents left. How much money had she at first?

LESSON IV.

DEVELOPMENT OF NUMBERS BY 5's.

1. 5+5+5+5+5+5+5+5+5+5+5+5=?
2. 4+5+5+5+5+5+5+5+5+5+5+5=?
3. 3+5+5+5+5+5+5+5+5+5+5+5=?
4. 2+5+5+5+5+5+5+5+5+5+5+5=?
5. 1+5+5+5+5+5+5+5+5+5+5+5=?
6. 1×5=? 2×5=? 3×5=? 4×5=? 5×5=? 6×5=? 7×5=? 8×5=? 9×5=? 10×5=? 11×5=? 12×5=?
7. 60−5−5−5−5−5−5−5−5−5−5−5=?
8. 59−5−5−5−5−5−5−5−5−5−5−5=?
9. 58−5−5−5−5−5−5−5−5−5−5−5=?
10. 57−5−5−5−5−5−5−5−5−5−5−5=?
11. 56−5−5−5−5−5−5−5−5−5−5−5=?
12. 5÷5=? 10÷5=? 15÷5=? 20÷5=? 25÷5=? 30÷5=? 35÷5=? 40÷5=? 45÷5=? 50÷5=? 55÷5=? 60÷5=?

---

13. 25 is 5 greater than what number? 5 less than what number?

14. 40 is 5 more than what number? 5 less than what number?

15. 35 is 5 times what number? 60 is 5 times what number?

16. What number is contained 5 times in 45? 5 times in 30?

17. What number is 5 less than 43? 5 more than 53?

18. What number is 5 less than 39? What number should be added to 54 to obtain 59?

19. How many times can 5 be taken from 30? 6 is one-fifth of what number?

20. 9 is one fifth of what number? Of what number is 10 one fifth?

21. 12 is one fifth of what number? What number should be added to 5 times 8 to obtain 45?

22. 5 times 7 and 4 equals what number? What four equal numbers equal 20?

23. What five equal numbers and 3 more equal 28? What nine equal numbers and 4 more equal 49?

24. 36 is equal to what seven equal numbers and one more? 27 is equal to what five equal numbers and 2 more?

---

25. Lucy took to market 50 cents, and spent 5 cents for lettuce. How many cents had she left?

26. Howard owned 35 cents, and his mother gave him 5 cents more. How many cents did he then have?

27. If one yard of velvet costs 7 dollars, how much will 5 yards cost?

28. James is 9 years old, and his mother is 5 times as old. How old is the mother?

29. A gentleman paid 11 dollars for a gun, and five times as much and 4 dollars more for a wagon. How much was the wagon worth?

30. If a boy can gather 5 quarts of nuts in one day, in how many days can he gather 40 quarts, at the same rate?

31. How many quarts of berries, at 5 cents a quart, may be bought for 55 cents?

32. A lady paid one fifth of 60 dollars for a shawl. How much did the shawl cost?

33. How many five-cent pieces are equal to 45 cents?

LESSON V.

DEVELOPMENT OF NUMBERS BY 6's.

1. 6+6+6+6+6+6+6+6+6+6+6+6=?
2. 5+6+6+6+6+6+6+6+6+6+6+6=?
3. 4+6+6+6+6+6+6+6+6+6+6+6=?
4. 3+6+6+6+6+6+6+6+6+6+6+6=?
5. 2+6+6+6+6+6+6+6+6+6+6+6=?
6. 1+6+6+6+6+6+6+6+6+6+6+6=?
7. 1×6=? 2×6=? 3×6=? 4×6=? 5×6=? 6×6=? 7×6=? 8×6=? 9×6=? 10×6=? 11×6=? 12×6=?
8. 72−6−6−6−6−6−6−6−6−6−6−6=?
9. 71−6−6−6−6−6−6−6−6−6−6−6=?
10. 70−6−6−6−6−6−6−6−6−6−6−6=?
11. 69−6−6−6−6−6−6−6−6−6−6−6=?
12. 68−6−6−6−6−6−6−6−6−6−6−6=?
13. 67−6−6−6−6−6−6−6−6−6−6−6=?
14. 6÷6=? 12÷6=? 18÷6=? 24÷6=? 30÷6=? 36÷6=? 42÷6=? 48÷6=? 54÷6=? 60÷6=? 66÷6=? 72÷6=?

15. 36 is 6 greater than what number? 6 less than what number?

16. 58 is six more than what number? 6 less than what number?

17. 42 is 6 times what number? 66 is 6 times what number?

18. What number is contained 6 times in 48? 6 times in 72?

19. What number is 6 less than 65? 6 more than 47?

20. What number is 6 less than 37? By what number should 43 be increased to obtain 49?

21. How many times can 6 be taken from 24? 8 is one sixth of what number?

22. 5 is one sixth of what number? Of what number is 9 one sixth?

23. 12 is one sixth of what number? By what number should 6 times 10 be increased to obtain 66?

24. 6 times 7 and 5 more equals what number? What three equal numbers equal 18?

25. What four equal numbers and 3 more equal 27? 50 is equal to what 8 equal numbers and 2 more?

26. 57 is equal to how many 6's and 3 more? 70 is equal to how many 6's and 4 more?

---

27. Frank received for a gift 55 cents, and expended 6 cents for an orange. How many cents had he left?

28. Sarah earned 48 cents by sewing, and 6 cents more by knitting. How much did she earn in all?

29. If one barrel of flour is worth 8 dollars, how many dollars are 6 barrels worth?

30. A boy can walk 9 rods a minute, and a dog can run 6 times as far in the same time. How far can the dog run in one minute?

31. 72 dollars is 6 times what a gentleman paid for a sheep. How much was paid for the sheep?

32. One sixth of 48 dollars is the price of one yard of velvet. How much is the velvet worth?

33. A paid 7 dollars for a vest, and 6 times as much and 5 dollars more for a coat. How much did they both cost him?

34. How much is paid for a cow that costs 11 times 6 dollars and 4 dollars more?

LESSON VI.

DEVELOPMENT OF NUMBERS BY 7's.

1. 7+7+7+7+7+7+7+7+7+7+7+7+7=?
2. 6+7+7+7+7+7+7+7+7+7+7+7+7=?
3. 5+7+7+7+7+7+7+7+7+7+7+7+7=?
4. 4+7+7+7+7+7+7+7+7+7+7+7+7=?
5. 3+7+7+7+7+7+7+7+7+7+7+7+7=?
6. 2+7+7+7+7+7+7+7+7+7+7+7+7=?
7. 1+7+7+7+7+7+7+7+7+7+7+7+7=?
8. 1×7=? 2×7=? 3×7=? 4×7=? 5×7=? 6×7=? 7×7=? 8×7=? 9×7=? 10×7=? 11×7=? 12×7=?
9. 84−7−7−7−7−7−7−7−7−7−7−7−7=?
10. 83−7−7−7−7−7−7−7−7−7−7−7−7=?
11. 82−7−7−7−7−7−7−7−7−7−7−7−7=?
12. 81−7−7−7−7−7−7−7−7−7−7−7−7=?
13. 80−7−7−7−7−7−7−7−7−7−7−7−7=?
14. 79−7−7−7−7−7−7−7−7−7−7−7−7=?
15. 78−7−7−7−7−7−7−7−7−7−7−7−7=?
16. 7÷7=? 14÷7=? 21÷7=? 28÷7=? 35÷7=? 42÷7=? 49÷7=? 56÷7=? 63÷7=? 70÷7=? 77÷7=? 84÷7=?

17. 56 is 7 more than what number? 7 less than what number?

18. 78 is 7 greater than what number? 7 less than what number?

19. 49 is 7 times what number? 63 is 7 times what number?

20. What number is contained 7 times in 56? In 84?

21. What number is 7 less than 75? 7 greater than 64?

22. By what number should 65 be increased to obtain 72? How many times can 7 be taken from 35?

23. 6 is one seventh of what number? 9 is one seventh of what number?

24. 12 is one seventh of what number? By what number should 8 times 7 be increased to obtain 63?

25. 7 times 7 and 6 more equals what number? What 4 equal numbers and 4 more equal 32?

26. What 6 equal numbers equal 42? What 5 equal numbers and 3 more equal 38?

27. 75 is equal to what 10 equal numbers and 5 more? 67 is equal to how many 7's and 4 more?

28. 83 is equal to how many 7's and 6 more? One seventh of 84 equals what number?

---

29. Clara bought a paper of 72 pins, and gave away 7. How many pins had she left?

30. 59 dollars is 7 dollars less than what was paid for a sleigh. How much was paid for the sleigh?

31. If one bushel of chestnuts is worth 7 dollars, how much are 9 bushels worth?

32. How many barrels of flour, at 7 dollars a barrel, may be bought for 56 dollars?

33. How much is paid for a quantity of corn that costs 11 times 7 dollars and 5 dollars more?

34. 63 acres is 7 times what a farmer has planted in wheat. How many acres of wheat has he?

35. One seventh of 77 years is George's age. How old is George?

36. There are 12 inches in one foot. How many inches are there in a line 7 feet long?

37. How much is the rent of a house a month for which there is paid 9 times 7 dollars and 6 dollars more?

38. 61 dollars is 5 dollars more than what 7 tons of coal cost. What is the price of one ton?

LESSON VII.

DEVELOPMENT OF NUMBERS BY 8's.

1. 8+8+8+8+8+8+8+8+8+8+8+8=?
2. 7+8+8+8+8+8+8+8+8+8+8+8=?
3. 6+8+8+8+8+8+8+8+8+8+8+8=?
4. 5+8+8+8+8+8+8+8+8+8+8+8=?
5. 4+8+8+8+8+8+8+8+8+8+8+8=?
6. 3+8+8+8+8+8+8+8+8+8+8+8=?
7. 2+8+8+8+8+8+8+8+8+8+8+8=?
8. 1+8+8+8+8+8+8+8+8+8+8+8=?
9. 1×8=? 2×8=? 3×8=? 4×8=? 5×8=?
6×8=? 7×8=? 8×8=? 9×8=? 10×8=?
11×8=? 12×8=?
10. 96−8−8−8−8−8−8−8−8−8−8−8=?
11. 95−8−8−8−8−8−8−8−8−8−8−8=?
12. 94−8−8−8−8−8−8−8−8−8−8−8=?
13. 93−8−8−8−8−8−8−8−8−8−8−8=?
14. 92−8−8−8−8−8−8−8−8−8−8−8=?
15. 91−8−8−8−8−8−8−8−8−8−8−8=?
16. 90−8−8−8−8−8−8−8−8−8−8−8=?
17. 89−8−8−8−8−8−8−8−8−8−8−8=?
18. 8÷8=? 16÷8=? 24÷8=? 32÷8=? 40÷8=?
48÷8=? 56÷8=? 64÷8=? 72÷8=? 80÷8=?
88÷8=? 96÷8=?

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19. 48 is 8 more than what number? 8 less than what number?

20. 57 is 8 greater than what number? 8 less than what number?

21. 5 times 8 equals what number? 8 times 9 equals what number?

22. What number is contained 8 times in 56? In 64?

23. By what number should 53 be increased to obtain 61? What number is 8 more than 49?

24. What number is 8 less than 77? 96 equals 8 times what number?

25. What number equals 8 times 7 and 6 more? What number equals 7 more than 8 times 11?

26. How many times can 8 be taken from 48? 9 equals one eighth of what number?

27. 12 equals one eighth of what number? Of what number is 8 one eighth?

28. One eighth of 48 is 3 less than one eighth of what number? 6 times 8 and 7 more equals what number?

29. What four equal numbers and 5 more equal 37? What 6 equal numbers are contained in 48?

30. What 8 equal numbers and 5 more equal 69? What 7 equal numbers are 6 less than 62?

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31. Horace in one day earned 56 cents, and gave 8 of them for a lead pencil. How much had he left?

32. Julia planted 65 seeds, and Mary planted 8 less than Julia. How many did Mary plant?

33. There are 9 square feet in one square yard. How many square feet are there in 8 square yards?

34. If a person work 8 hours each day, how many hours will he work in 12 days?

35. A boy was asked how many marbles he had, and replied that if he had 8 times as many, he would have 96. How many marbles had he?

36. If a ship sails 8 miles an hour, in what time, at the same rate, will she sail 88 miles?

37. One-eighth of 72 dollars is what a gentleman paid for 2 tickets to the opera. How much did they cost?

LESSON VIII.

DEVELOPMENT OF NUMBERS BY 9's.

1. 9+9+9+9+9+9+9+9+9+9+9+9=?
2. 8+9+9+9+9+9+9+9+9+9+9+9=?
3. 7+9+9+9+9+9+9+9+9+9+9+9=?
4. 6+9+9+9+9+9+9+9+9+9+9+9=?
5. 5+9+9+9+9+9+9+9+9+9+9+9=?
6. 4+9+9+9+9+9+9+9+9+9+9+9=?
7. 3+9+9+9+9+9+9+9+9+9+9+9=?
8. 2+9+9+9+9+9+9+9+9+9+9+9=?
9. 1+9+9+9+9+9+9+9+9+9+9+9=?
10. 1×9=? 2×9=? 3×9=? 4×9=? 5×9=? 6×9=? 7×9=? 8×9=? 9×9=? 10×9=? 11×9=? 12×9=?
11. 108−9−9−9−9−9−9−9−9−9−9−9=?
12. 107−9−9−9−9−9−9−9−9−9−9−9=?
13. 106−9−9−9−9−9−9−9−9−9−9−9=?
14. 105−9−9−9−9−9−9−9−9−9−9−9=?
15. 104−9−9−9−9−9−9−9−9−9−9−9=?
16. 103−9−9−9−9−9−9−9−9−9−9−9=?
17. 102−9−9−9−9−9−9−9−9−9−9−9=?
18. 101−9−9−9−9−9−9−9−9−9−9−9=?
19. 100−9−9−9−9−9−9−9−9−9−9−9=?
20. 9÷9=? 18÷9=? 27÷9=? 36÷9=? 45÷9=? 54÷9=? 63÷9=? 72÷9=? 81÷9=? 90÷9=? 99÷9=? 108÷9=?

21. 45 is 9 more than what number? 9 less than what unmber?

22. 76 is 9 greater than what number? 9 less than what number?

23. 6 times 9 equals what number? 9 times 7 equals what number?

24. What number is contained 9 times in 72? In 108?

25. By what number should 58 be increased to obtain 67? What number is 9 more than 85?

26. What number is 9 less than 97? 81 equals 9 times what number?

27. What number equals 9 times 6 and 7 more? How many times can 9 be taken from 54?

28. 8 equals one ninth of what number? 11 equals one ninth of what number?

29. Of what number is 10 one ninth? One ninth of 81 equals what number?

30. 7 times 9 equals 7 less than what number? What 5 equal numbers are contained in 45?

31. What 7 equal numbers and 6 more equal 69? What 8 equal numbers are 5 less than 77?

32. What 9 equal numbers and 7 more equal 88? One ninth of 45 is 4 less than one ninth of what number?

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33. James has 69 pennies in one box, and 9 pennies in another. How many pennies are there in the two boxes?

34. Mary had 76 pinks in her garden, and 9 of them were destroyed. How many had she left?

35. A farmer has 12 sheep in the barn, and 9 times as many in the field. How many sheep are in the field?

36. How many more are there in the field than in the barn?

37. Of the pupils of a school 6 were absent, and 9 times as many were present. How many were present? How many belonged to the school?

38. At 9 cents a pound, how much will 8 pounds of sugar cost?

39. At 9 shillings a bushel, how many bushels of potatoes may be bought for 99 shillings?

40. If 9 men can pick 108 bushels of apples in one day, how many bushels can one man pick?

41. What is one yard of velvet worth if 9 yards are worth 72 dollars?

42. One ninth of 63 dollars is what John paid for a hat. What did the hat cost?

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43. Jane has 11 dollars, and her father has 9 times as much and 8 dollars more. How many dollars has her father? How many dollars have both?

44. Jennie's grandfather is 72 years of age, and she is one ninth as old. How old is Jennie? In how many more years will she be 17 years old?

45. A gentleman spent 45 dollars in purchase of cloth at 5 dollars a yard. How many yards did he buy? How many dollars had he left?

46. At 6 dollars a ton, how many tons of coal can I buy if I have 76 dollars? How many dollars will I have left?

47. One seventh of 84 dollars is 3 times the cost of my chair. How much did my chair cost?

48. Sold 12 barrels of flour, worth 8 dollars a barrel, for 89 dollars. How many dollars did I lose?

49. How much more than 96 dollars should a gentleman have in order to buy 12 cords of wood at 9 dollars a cord?

50. A boy walked 12 days, at the rate of 9 miles a day. How many miles farther should he walk so as to travel 120 miles?

51. A gentleman earned 9 dollars a day for 8 days, and spent 8 dollars a day for 8 days. How much has he left?

52. Bought 8 yards of cloth at 12 dollars a yard, and gave in payment 10 ten-dollar bills. How much change should I receive?

LESSON IX.

PRACTICE EXERCISES.

This lesson should be studied as follows: Use first the sign +, as 11+2=13, etc.; then the sign −, as 11−2=9, etc.; afterwards both signs, as 11±2=13 or 9.

1.	11±2=?	21±2=?	31±2=?	41±2=?	etc.
2.	11±3=?	21±3=?	31±3=?	41±3=?	etc.
3.	11±4=?	21±4=?	31±4=?	41±4=?	etc.
4.	11±5=?	21±5=?	31±5=?	41±5=?	etc.
5.	11±6=?	21±6=?	31±6=?	41±6=?	etc.
6.	11±7=?	21±7=?	31±7=?	41±7=?	etc.
7.	11±8=?	21±8=?	31±8=?	41±8=?	etc.
8.	11±9=?	21±9=?	31±9=?	41±9=?	etc.
9.	12±2=?	22±2=?	32±2=?	42±2=?	etc.
10.	12±3=?	22±3=?	32±3=?	42±3=?	etc.
11.	12±4=?	22±4=?	32±4=?	42±4=?	etc.
12.	12±5=?	22±5=?	32±5=?	42±5=?	etc.
13.	12±6=?	22±6=?	32±6=?	42±6=?	etc.
14.	12±7=?	22±7=?	32±7=?	42±7=?	etc.
15.	12±8=?	22±8=?	32±8=?	42±8=?	etc.
16.	12±9=?	22±9=?	32±9=?	42±9=?	etc.
17.	13±2=?	23±2=?	33±2=?	43±2=?	etc.
18.	13±3=?	23±3=?	33±3=?	43±3=?	etc.
19.	13±4=?	23±4=?	33±4=?	43±4=?	etc.
20.	13±5=?	23±5=?	33±5=?	43±5=?	etc.
21.	13±6=?	23±6=?	33±6=?	43±6=?	etc.
22.	13±7=?	23±7=?	33±7=?	43±7=?	etc.
23.	13±8=?	23±8=?	33±8=?	43±8=?	etc.
24.	13±9=?	23±9=?	33±9=?	43±9=?	etc.

25.	$14\pm2=?$	$24\pm2=?$	$34\pm2=?$	$44\pm2=?$	etc.
26.	$14\pm3=?$	$24\pm3=?$	$34\pm3=?$	$44\pm3=?$	etc.
27.	$14\pm4=?$	$24\pm4=?$	$34\pm4=?$	$44\pm4=?$	etc.
28.	$14\pm5=?$	$24\pm5=?$	$34\pm5=?$	$44\pm5=?$	etc.
29.	$14\pm6=?$	$24\pm6=?$	$34\pm6=?$	$44\pm6=?$	etc.
30.	$14\pm7=?$	$24\pm7=?$	$34\pm7=?$	$44\pm7=?$	etc.
31.	$14\pm8=?$	$24\pm8=?$	$34\pm8=?$	$44\pm8=?$	etc.
32.	$14\pm9=?$	$24\pm9=?$	$34\pm9=?$	$44\pm9=?$	etc.
33.	$15\pm2=?$	$25\pm2=?$	$35\pm2=?$	$45\pm2=?$	etc.
34.	$15\pm3=?$	$25\pm3=?$	$35\pm3=?$	$45\pm3=?$	etc.
35.	$15\pm4=?$	$25\pm4=?$	$35\pm4=?$	$45\pm4=?$	etc.
36.	$15\pm5=?$	$25\pm5=?$	$35\pm5=?$	$45\pm5=?$	etc.
37.	$15\pm6=?$	$25\pm6=?$	$35\pm6=?$	$45\pm6=?$	etc.
38.	$15\pm7=?$	$25\pm7=?$	$35\pm7=?$	$45\pm7=?$	etc.
39.	$15\pm8=?$	$25\pm8=?$	$35\pm8=?$	$45\pm8=?$	etc.
40.	$15\pm9=?$	$25\pm9=?$	$35\pm9=?$	$45\pm9=?$	etc.
41.	$16\pm2=?$	$26\pm2=?$	$36\pm2=?$	$46\pm2=?$	etc.
42.	$16\pm3=?$	$26\pm3=?$	$36\pm3=?$	$46\pm3=?$	etc.
43.	$16\pm4=?$	$26\pm4=?$	$36\pm4=?$	$46\pm4=?$	etc.
44.	$16\pm5=?$	$26\pm5=?$	$36\pm5=?$	$46\pm5=?$	etc.
45.	$16\pm6=?$	$26\pm6=?$	$36\pm6=?$	$46\pm6=?$	etc.
46.	$16\pm7=?$	$26\pm7=?$	$36\pm7=?$	$46\pm7=?$	etc.
47.	$16\pm8=?$	$26\pm8=?$	$36\pm8=?$	$46\pm8=?$	etc.
48.	$16\pm9=?$	$26\pm9=?$	$36\pm9=?$	$46\pm9=?$	etc.
49.	$17\pm2=?$	$27\pm2=?$	$37\pm2=?$	$47\pm2=?$	etc.
50.	$17\pm3=?$	$27\pm3=?$	$37\pm3=?$	$47\pm3=?$	etc.
51.	$17\pm4=?$	$27\pm4=?$	$37\pm4=?$	$47\pm4=?$	etc.
52.	$17\pm5=?$	$27\pm5=?$	$37\pm5=?$	$47\pm5=?$	etc.
53.	$17\pm6=?$	$27\pm6=?$	$37\pm6=?$	$47\pm6=?$	etc.
54.	$17\pm7=?$	$27\pm7=?$	$37\pm7=?$	$47\pm7=?$	etc.
55.	$17\pm8=?$	$27\pm8=?$	$37\pm8=?$	$47\pm8=?$	etc.
56.	$17\pm9=?$	$27\pm9=?$	$37\pm9=?$	$47\pm9=?$	etc.

57. $18\pm2=?$ $28\pm2=?$ $38\pm2=?$ $48\pm2=?$ etc.
58. $18\pm3=?$ $28\pm3=?$ $38\pm3=?$ $48\pm3=?$ etc.
59. $18\pm4=?$ $28\pm4=?$ $38\pm4=?$ $48\pm4=?$ etc.
60. $18\pm5=?$ $28\pm5=?$ $38\pm5=?$ $48\pm5=?$ etc.
61. $18\pm6=?$ $28\pm6=?$ $38\pm6=?$ $48\pm6=?$ etc.
62. $18\pm7=?$ $28\pm7=?$ $38\pm7=?$ $48\pm7=?$ etc.
63. $18\pm8=?$ $28\pm8=?$ $38\pm8=?$ $48\pm8=?$ etc.
64. $18\pm9=?$ $28\pm9=?$ $38\pm9=?$ $48\pm9=?$ etc.

65. $19\pm2=?$ $29\pm2=?$ $39\pm2=?$ $49\pm2=?$ etc.
66. $19\pm3=?$ $29\pm3=?$ $39\pm3=?$ $49\pm3=?$ etc.
67. $19\pm4=?$ $29\pm4=?$ $39\pm4=?$ $49\pm4=?$ etc.
68. $19\pm5=?$ $29\pm5=?$ $39\pm5=?$ $49\pm5=?$ etc.
69. $19\pm6=?$ $29\pm6=?$ $39\pm6=?$ $49\pm6=?$ etc.
70. $19\pm7=?$ $29\pm7=?$ $39\pm7=?$ $49\pm7=?$ etc.
71. $19\pm8=?$ $29\pm8=?$ $39\pm8=?$ $49\pm8=?$ etc.
72. $19\pm9=?$ $29\pm9=?$ $39\pm9=?$ $49\pm9=?$ etc.

SECTION I.

DEFINITIONS.

Article 1. A ***Unit*** is a single thing, or one of any kind.

Thus, one, one hat, one gallon, one year.

One denotes a single thing; one and one more is called *Two;* two and one more is called *Three;* and thus, by the successive additions of one, are obtained the several numbers *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*.

2. A ***Number*** is an expression of one or more units.

Thus, one, four, six miles, fifty dollars, one hundred, are numbers.

3. ***Arithmetic*** is that branch of mathematics which treats of numbers and their applications.

Hence, all forms of counting and business operations involving the use of numbers belong to Arithmetic.

4. A ***Denominate Number*** is a number whose kind of unit is named.

Thus, 2 apples, 5 bushels, 30 yards, are denominate numbers.

5. An ***Abstract Number*** is a number whose kind of unit is not named.

Thus, 4, 7, 40, 100, are abstract numbers.

6. The ***Unit*** or ***Unit Value*** of a number is *one* of that number.

Thus, 1 foot is the unit value of 5 feet or 10 feet; 1 is the unit value of 5, 10 or 50.

7. ***Like Numbers*** are numbers that have equal unit values.

Thus, 2 miles and 8 miles are like numbers, because they have equal unit values, 1 mile, 1 mile.

8. ***Unlike Numbers*** are numbers that have different unit values.

Thus, 5 boys, 10 apples, 20, 30 dollars, are unlike numbers, because their unit values, 1 boy, 1 apple, 1, 1 dollar, are not alike.

9. Units are of the *same kind*, when they are either abstract or of the same denomination.

Thus, 2, 4, 10, 50, express units of the same kind.
So, 2 feet, 4 feet, 10 feet, 50 feet.

10. Units are of different kinds when they are of different denominations.

Thus, 3 dollars, 4 gallons, 2 quarts, express units of different kinds.

11. ***Simple Numbers*** are numbers that express units of the same kind.

Thus, 4, 6, 12, 25, 48, are simple numbers.
So, 3 years, 40 years, 100 years. Hence,

Simple numbers are either abstract or denominate.

12. ***Compound Numbers*** are numbers that, regarded as one quantity, express units of different kinds.

Thus, 12 feet 4 inches, 8 pounds 6 ounces, are compound numbers. Hence,

Compound numbers are always denominate.

13. An ***Integer*** or ***Integral Number*** is a number that expresses one or more whole units.

Thus, 7, 16 pears, 100, are integers.

14. ***Figures*** are characters used to represent numbers.

SECTION II.

NOTATION AND NUMERATION.

Art. 15. Notation is a method of representing numbers by written characters. There are two methods of notation in ordinary use.

1. The ***Roman Notation,*** which employs *letters.*
2. The ***Arabic Notation,*** which employs *figures.*

The letters employed in the *Roman Notation* are the seven capital letters, I, V, X, L, C, D, M.

I=1. V=5. X=10. L=50. C=100. D=500. M=1000.

Other values are represented by certain combinations of these letters in accordance with the following

PRINCIPLES.

1. If a letter is repeated, or if a letter or combination of letters of less value follows a letter of greater value, the sum of the values is the value of the combination.

Thus, III=3. XXX=30. VI=6. DC=600. XIX=19.

2. If a letter of greater value follows a letter of less value, the difference of the two values is the value of the combination.

Thus, IX=9. XL=40. CD=400.

3. A dash (—) placed over a letter or combination of letters, gives it a thousand-fold value.

Thus, $\overline{\text{I}}$=one thousand. $\overline{\text{XX}}$=twenty thousand.

16. The figures employed in the *Arabic Notation* are the ten figures, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Other values are represented by these figures, in certain simple combinations, in accordance with principles explained in Art. 17.

The first of these figures is called *zero*, *naught*, or *cipher*, and represents no value, for its use implies the absence of number. The remaining nine figures are called *digits* or *significant figures*, representing *one*, *two*, *three*, etc., units.

17. *Orders of Figures* are represented in the following manner. A figure in the first or right hand place is a *figure of the first order;* in the second place, *a figure of the second order;* in the third place, *a figure of the third order:* and so on, for all successive orders.

Thus, in the number 4765, the 5 is a figure of the first order; the 6 is a figure of the second order; the 7 is a figure of the third order; and the 4 is a figure of the fourth order.

PRINCIPLES.

1. In counting, *one, or a single thing*, is the basis of all numbers, and is called *unit*, or first order.

2. *Ten single things*, or *units*, make one *group* or *collection*, called *tens*, or second order.

3. *Ten of the collections called tens* make one *group* or *collection* called *hundreds*, or third order.

4. Ten of the collections called hundreds make one *group* or *collection* called *thousands*, or fourth order. And similarly, *ten collections of any order* make *one collection* of an order of the next larger denomination. The first ten of these orders are named as follows: Units, Tens, Hundreds, Thousands, Ten-thousands, Hundred-thousands, Millions, Ten-millions, Hundred-millions, Billions.

The first three of these orders, *units*, *tens* and *hundreds*, form a group called the ***Period of Units;*** the second three, in like manner, form a group called the ***Period of***

Thousands, and in like manner, the successive higher periods, of ***Millions, Billions, Trillions,*** etc.

Thus, 345628 is a number composed of 345 thousands, 628 units, and is read 345 thousand 628.

18. Figures, it is plain, may therefore represent two values, viz.: Absolute and Local.

Absolute Value is represented by a figure, or by a combination of figures, standing alone.

Thus, 4, 6, 8, 24, 149, represent absolute value.

Local Value of figures, whether represented by a figure or combination of figures, is the value as determined by the place they occupy.

Thus, in 576, the 6 of the first order, the 7 of the second, and the 5 of the third order, each represent a local value.

So, the 76 and the 57.

19. Numeration is the method of reading numbers.

Thus, 2786 is read two thousand, seven hundred, eighty-six: or, six units, eight tens (eighty), seven hundreds, and two thousands.

20. Rule for Notation.—*Begin at the highest or lowest order, and write in each successive order the figures belonging to that order, filling the vacant orders with ciphers.*

Rule for Numeration.—*Begin at the highest or lowest order, and read the successive orders of figures, giving the name of each period, except the period of units.*

21. TABLE OF NOTATION AND NUMERATION.

	5th Period.			4th Period.			3d Period.			2d Period.			1st Period.		
Names of Periods.	Trillions.			Billions.			Millions.			Thousands.			Units.		
Names of Orders.	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
Number.	3	7	5 ,	4	6	2 ,	3	7	5 ,	4	6	3 ,	7	2	5

ORDERS OF FIGURES LOWER THAN UNITS.

22. As orders of figures represent values of units, and values higher than units, so may they represent values lower than units, as

In all orders of figures a figure written in one order represents a value one tenth of the value represented by the same figure one order to the left; so a figure written next to the right of units represents a value one tenth of the value represented by the same figure in the order of units. Tenths of units are tenths. Therefore, the order occupied by the figures written next to the right of units is called the order of *tenths*, and is always separated from the order of units by a mark called the ***Decimal Point.***

23. The order occupied by the figure written next to the right of tenths is called *hundredths*, and the order occupied by the figure written next to the right of hundredths is called *thousandths*. These three orders of *tenths*, *hundredths*, and *thousandths*, form a group of orders called the ***Period of Thousandths.***

Thus, 245.364 is a number composed of 245 *units* and 364 *thousandths*, or of two periods, a period of units and a period of thousandths: and is read two hundred forty-five and three hundred sixty-four thousandths.

24. It will be observed that the principles governing the order of units and those that are higher, have equal application to the *orders lower than units;* that is, *they are written in the scale of ten.* Those orders lower than units are called *Decimals*, and all operations performed upon them are identical with corresponding operations upon integral numbers.

25. ***United States Money*** is written in accordance with this feature of notation. The order of units and those that are higher are represented by dollars, the *tenths* and *hundredths* by cents, and the *thousandths* by mills.

Thus, $3.375 is read three dollars, thirty-seven cents and five mills.
$5.098 is read five dollars, nine cents and eight mills; and
$8.406 is read eight dollars, forty cents and six mills.

26. The character $ means dollars, and is written at the left of the number, and is supposed to be a monogram of the letters U. S.

The *cents* and *mills* may be read as a *decimal* of the dollar, or the mills as *decimal* of the cent.

Thus, $6.236 may be read *six and two hundred thirty six thousandths dollars*, or it may be read *six dollars, twenty-three and six-tenths cents.*

27. DECIMAL NOTATION AND UNITED STATES MONEY COMPARED.

Decimal Notation.	***U. S. Money.***
56.20 is read *fifty-six and two tenths, or twenty hundredths.*	$56.20 is read *fifty-six dollars and twenty cents.*
63.45 is read *sixty-three and forty-five hundredths.*	$63.45 is read *sixty-three dollars and forty-five cents.*
75.356 is read *seventy-five and three hundred fifty-six thousandths.*	$75.356 is read *seventy-five dollars, thirty-five cents and six mills.*

SLATE EXERCISES.

Write in figures:

1. Seven.
2. Twelve.
3. Seventeen.
4. Twenty.
5. Thirty-five.
6. Fifty-nine.
7. Ninety-seven.
8. Two hundred nineteen.
9. Three hundred eighty-six.
10. Six hundred three.
11. Nine hundred forty.
12. One thousand, six hundred fifty-four.
13. Four thousand, nine hundred sixteen.
14. Seven thousand, seventy-nine.

15. Nine thousand, eight.

16. Twelve thousand, nineteen.

17. Sixteen thousand, four hundred seventy-six.

18. Thirty-five thousand, two hundred eleven.

19. Fifty-four thousand, three hundred ninety-seven.

20. Sixty-six thousand, six hundred sixty-six.

21. Ninety-four thousand, eighteen.

22. One hundred twenty-five thousand, two hundred sixteen.

23. Two hundred eleven thousand, nine hundred eleven.

24. Six hundred three thousand, eight hundred eight.

25. Nine hundred seventy-five thousand, sixty.

26. One million, three hundred twenty-six thousand, four hundred seventy-nine.

27. Two million, seven hundred thousand, five hundred ninety-eight.

28. Three million, sixty-four thousand, two hundred eighty-one.

29. Four million, four thousand, four hundred.

30. Five million, five thousand, five.

31. Seventy-two million, nine hundred forty-five thousand, eight hundred sixteen.

32. Forty-seven million, fifty-nine thousand, six hundred fifty-four.

33. Sixty million, sixty thousand, six hundred.

34. Eighty million, eighty thousand, eighty.

35. Thirty-nine million, eight thousand, nine.

36. Fifty-six million, three hundred twenty thousand, nine hundred sixteen.

37. Twenty-nine million, one hundred seven thousand, ninety-four.

38. Seventy-five million, twenty-eight thousand, nine.

39. Two hundred forty-seven million, six hundred twenty-five thousand, eight hundred sixteen.

40. Five hundred forty million, three hundred eighty-seven thousand, twenty-one.

41. Six hundred three million, eighty-nine thousand, seven.

42. Seven hundred fifty-six million, eight thousand, forty.

43. Eight hundred million, eight hundred thousand, eight hundred.

44. Nine hundred ninety-nine million, nine thousand, nine.

Write in words, read from the page, or write in figures from dictation.

1.	17	*28.*	8036	*55.*	5120674
2.	26	*29.*	9003	*56.*	6204638
3.	38	*30.*	10600	*57.*	7035690
4.	43	*31.*	10128	*58.*	8002637
5.	51	*32.*	25364	*59.*	9200075
6.	67	*33.*	37428	*60.*	10000000
7.	76	*34.*	45546	*61.*	18124620
8.	84	*35.*	52360	*62.*	23742635
9.	95	*36.*	64208	*63.*	35428640
10.	100	*37.*	73064	*64.*	46375904
11.	119	*38.*	80975	*65.*	57298027
12.	236	*39.*	93602	*66.*	69380324
13.	328	*40.*	100000	*67.*	72506241
14.	456	*41.*	123250	*68.*	83037256
15.	508	*42.*	267580	*69.*	90364562
16.	643	*43.*	329705	*70.*	100000000
17.	759	*44.*	463076	*71.*	123234345
18.	804	*45.*	520362	*72.*	231456213
19.	973	*46.*	605729	*73.*	325368450
20.	1000	*47.*	750038	*74.*	432523609
21.	1325	*48.*	800201	*75.*	561431035
22.	2140	*49.*	900037	*76.*	632520362
23.	3254	*50.*	1000000	*77.*	775306728
24.	4562	*51.*	1234526	*78.*	821078584
25.	5674	*52.*	2362340	*79.*	950364260
26.	6307	*53.*	3798709	*80.*	1000000000
27.	7040	*54.*	4569044	*81.*	9909909909

REVIEW QUESTIONS.

What is a Unit? Give example. What does *one* denote? How find the several successive numbers from one? What is a Number? Give example. What is Arithmetic? Denominate Number? Abstract Number? Give examples of each. What is the Unit of a Number? Illustrate. What are Like Numbers? Unlike Numbers? Give examples of both. When are Units of the same kind? Examples. When are Units of different kinds? Examples. What are Simple Numbers? Give examples. To which kind of numbers do Simple Numbers belong? What are Compound Numbers? To which kind of numbers do Compound Numbers belong? What is an Integer? Example. What is a figure?

What is Notation? How many methods of Numerical Notation? What kind of characters are used by each? Name the characters used in the Roman Notation, and their values. What is the first principle that governs the combinations of these letters to express number? Second principle? Third principle? Name the characters employed in the Arabic Notation. What is the use of the Cipher? What are the other nine figures called? What do they severally represent? How are *Orders* of figures represented? Illustrate by an example. What is the first principle governing the combination of figures to express number? The second principle? The third principle? The fourth principle? Name the first ten of these orders. How many orders are used to form one period? Name the first five periods. How is absolute value represented? Local value? Give examples of each.

What is Numeration? Give rule for Notation. Rule for Numeration. Name the order of figures next to the right of units. What is the second order below units called? The third order? What group do these three orders form? What separates the order of tenths from the order of units? What name is given to the orders lower than units? In what scale of numbers are they written? What are the names used to express United States Money? Which of these correspond to the order of units and those orders that are higher? To tenths and hundredths? To thousandths? What does the character $ signify? How may *cents* and *mills* be read? *Mills?*

SECTION III.

ADDITION.

Art. 28. In 6 there are six units, and in 4 there are four units. In 6 and 4, which are 10, there are six units and four units, equal to ten units. 10 therefore contains as many units as 6 and 4, and is called the *sum* of 6 and 4.

10 is also the sum of 7 and 3, or 5 and 3 and 2, or of any two or more numbers that contain as many units. Hence,

29. The ***Sum*** of two or more numbers is the number that contains all their units and no more.

Thus, 9 is the sum of 4 and 5, or 6 and 3.

30. ***Addition*** is the process of obtaining the sum of two or more numbers.

31. The ***Sign*** of ***Addition*** is +, called *plus*. When placed between two numbers it indicates the addition of one number to the other.

Thus, 5+7 signifies that 7 is to be added to 5, or that 5 is to be added to 7.

32. ***Addends*** are the numbers to be added.

Thus, 5+4+3+8=20. 5, 4, 3, and 8, are addends.

33. The ***Sign*** of ***Equality*** is =, and is read *is equal to*, or *equals*, and when placed between two quantities it signifies that the quantity on one side is equal to the quantity on the other side.

Thus, 5+7=12; or, 4+5=3+6; or, 8=3+5.

PRINCIPLE.

Numbers can be added only to *like* numbers.

ORAL PROBLEMS.

1. A boy paid 25c. for a First Reader, and 10c. for a slate. How much did he pay for both?

SOLUTION.—Since he paid 25c. for a First Reader, and 10c. for a slate, he paid for both the sum of 25c. and 10c., which is 35c.

2. A girl paid 30c. for a Second Reader, 12c. for a writing-book, and 8c. for a sponge. How much did she pay for all?

3. 20c. was paid for a drawing-book, 15c. for a speller, and 9c. for a pencil. How much was paid for all?

4. John has 18 marbles, James 12 marbles, and George 9 marbles. How many have they all?

5. Mary had 32c., her father gave her 11 more, and her mother 8 more. How many did she then have?

6. How many bushels of apples will 3 men pick in one day, if the first pick 16 bushels, the second 12 bushels, and the third 11 bushels?

7. Minnie, Emma, and Jane went out berrying; Minnie gathered 21 pints, Emma 15 pints, and Jane 12 pints. How many pints did they all gather?

8. A gentleman has 12 books on one table, 10 on another, 9 on a third table, and 8 on the fourth. How many has he on the four tables?

9. A man sold 10 turkeys to one man, 9 to another, 8 to a third, and kept 7. How many turkeys had he at first?

10. A gentleman kept 15 sheep in one field, 9 in another, 8 in a third, and 7 in a fourth. How many sheep did he have?

11. A woman paid $12 for a shawl, $10 for a bonnet, $8 for a chain, and $6 for a pair of boots. How much money did she expend?

12. A gentleman bought a sleigh for $45, a set of harness for $15, a robe for $7, and a whip for $5. How much did he pay for all?

13. I saw 36 birds in one flock, 14 in another, 12 in a third, and 8 in a fourth. How many birds did I see?

14. A lady put up 30 quarts of cherries, 20 quarts of plums, 15 quarts of peaches, and 9 quarts of tomatoes. How many quarts of fruit did she put up?

15. A man went on a journey of four days; the first day he traveled 28 miles, the second 12 miles, the third 15 miles, and the fourth 9 miles. How far did he travel?

16. Bought 20 yards of muslin from one merchant, 20 yards from another, and 30 yards from a third. How many yards did I buy?

17. Gave $9 for a pair of boots, $8 for a coat, and 90c. for a pair of gloves. How much did they all cost?

18. A boy wrote 20 words in one minute, 16 the second, 10 the third, 8 the fourth, and 6 the fifth. How many words did he write in five minutes?

19. Gave $5 for a hat, 50c. for a neck-tie, $4 for a vest, and 40c. for a pair of cuffs. How much was given for all?

20. James earned $8, and William $7 in one week, and their father as much as both. How much did they all earn?

21. Paid 20c. for 12 oranges, and 30c. for 15 oranges. How much money did I expend? How many oranges did I buy?

22. Paid 50c. for 13 lemons, and 40c. for 9 lemons. How many lemons did I buy? How much did they cost?

23. John worked 7 days for $4, and James 8 days for $7, and Rufus as many days as John and James for $11. How many days did they all work? How much did they receive?

24. Sold a chain for $11, a watch for $9 more, and a cow for as much as both watch and chain. How much did I get for all?

25. Spent $5 Monday, $7 Tuesday, $9 Wednesday, $5 Thursday, $2 Friday, and 50c. Saturday. How much did I spend during the week?

26. Paid \$3 for 20 quarts of berries, \$2 for 15 quarts, and 60c. for 5 quarts. How many quarts of berries did I buy? How much did they cost?

27. Gave 30c. for 2 pounds of beef, 40c. for 3 pounds of pork, and 20c. for 3 pounds of mutton. How many pounds of meat did I buy? How much did I pay for meat?

28. A man bought a sleigh for \$20, a harness for \$15, and sold them at a gain of \$9. How much did he receive for them?

29. A cage was made of 12 wires on each side, 8 on each end, and 30 in the roof. How many wires were required to make the cage?

30. How many strokes does a clock make in 12 hours, that strikes the hours, and 1 each half hour?

31. A room is 20 feet long, and 15 feet wide. How many feet long is a line that would reach entirely around the room?

32. A gentleman upon a journey traveled for 6 days, increasing the distance traveled each day by 5 miles. If he traveled 10 miles the first day, how far did he travel during his journey?

33. Paid \$12 for books, \$18 for clothes, and as much for a horse as for both, plus \$10. How much money did I expend?

34. A person walked 20 miles from home in one day, and the second day 30 miles farther, the third day he rode the entire distance home again. How far did he travel during the three days?

35. Two houses are 6 miles apart. How far will that person travel who starts from one of them, visits the other, and returns 5 times?

36. In an orchard there are 12 cherry trees, 16 pear trees, and four more apple trees than pear trees. How many trees are there in the orchard?

37. A forester cut 15 cords of hickory wood, 20 cords of beech wood, 15 cords of oak wood, and 10 cords more of

maple wood than he cut of beech wood. How many cords of wood did he cut?

38. A man bought one lot of flour for $30, and another lot for $35. He sold the first lot at a gain of $10, and the second lot at a gain of $15. How much did he receive for both lots of flour?

39. I had 3 flocks of geese, the first containing 28, the second 22, and the third 25. I subsequently added 10 to each flock. How many geese did I then have?

40. A boy who had 4 bags, put into each one 15 nuts at one time, 10 into each at another time, and the third time he put 12 into each of three of them, and 14 into the fourth one. How many nuts are in the bags?

41. Bought 6 barrels of flour at $12 a barrel, and sold them at a gain of $9. How much did I receive for them?

ANALYSIS OF ADDITION.

42. What is the sum of 356+543+879?

PROCESS.

356
543
879
———
1778

ANALYSIS.—1. Write the addends so that figures of the same order shall stand in the same column.

2. Add the figures in the column of units; thus, 9, 12, 18 units, equal to 1 ten and 8 units. Write the 8 units in the order of units, and add the 1 ten to the column of tens; thus,

3. 1, 8, 12, 17 tens, equal to 1 hundred and 7 tens. Write the 7 tens in the order of tens, and add the 1 hundred to the column of hundreds; thus,

4. 1, 9, 14, 17 hundreds, equal to 1 thousand and 7 hundreds. Write the 7 hundreds in the order of hundreds, and the 1 thousand in the order of thousands.

The sum of 356+543+879 is therefore 1778.

NOTE 1. Pupils should not be permitted to acquire the habit of saying 9 and 3 are 12 and 6 are 18, etc. It is better to name results only, in each addition.

2. It will give greater facility in operations of addition, and lead to the detection of errors, to require pupils to add in a reverse order.

WRITTEN EXERCISES.

43. Add 9 thousand 3, 58 thousand 68, 64 thousand 208, 99 thousand 99, 273 thousand 574, 540 thousand 730, 879 thousand 7, 386.

44. Add 12 thousand 29, 79 thousand 524, 357 thousand 58, 792 thousand 218, 854 thousand 679, 927 thousand 9, 678 thousand 75, 2368.

45. Add 239 thousand 316, 528 thousand 97, 2 million 365 thousand 297, 7 million 67 thousand 954, 9 million 95 thousand 658, 8 million 586 thousand, 9 million 209.

46. Add 538 thousand 297, 3 million 3, 5 million 5 thousand 555, 6 million 72346, 9 million 75 thousand 654, 10 million 10 thousand 10, 12 million 807307.

47. Add 5 million 467 thousand 956, 729 thousand 475, 3 million 250 thousand 563, 17 million 368 thousand 845, 1729, 12 thousand 699, 4 million 37 thousand 76, 36 million 754 thousand 684, 9 million 500.

48. Add 236 million 827 thousand 973, 56 million 638, 829 thousand 976, 5 million 38 thousand 647, 798 thousand 364, 45 million 7, 7 thousand 8, 375 million 984 thousand 879, 99 million 99 thousand 99.

49. Add 3 thousand 709, 53 thousand 678, 4 million 579 thousand 863, 7 million 620 thousand 308, 495 thousand 678, 7 million 875 thousand 689, 29844.

50. Add 47854, 328 thousand 567, 2 million 47 thousand 854, 579 thousand 863, 4 million 876 thousand 594, 3 million 2 thousand 8, 97 thousand 582, 9010987.

51. Add 87 million 964 thousand 757, 3 million 986 thousand 759, 975 thousand 368, 77 thousand 675, 7 thousand 854, 989, 78, 9, 123674, 8478.

52. Add 43 thousand 687, 56 thousand 437, 64 thousand 482, 75 thousand 438, 79 thousand 347, 86 thousand 596, 27 thousand 368, 39 thousand 564.

53. Add 728 thousand 756, 689 thousand 476, 8 million

674 thousand 758, 12 million 846 thousand 378, 56 thousand 478, 9 thousand 64, 462 thousand 965, 778899.

54. Add 678 thousand 345, 756 thousand 987, 545 thousand 876, 787 thousand 456, 476 thousand 974, 856 thousand 358, 798 thousand 567, 534 thousand 765, 739 thousand 748, 593 thousand 584, 634 thousand 675, 587 thousand 378, 798 thousand 786.

55. Add 543 thousand 678, 789 thousand 657, 678 thousand 545, 654 thousand 787, 479 thousand 674, 853 thousand 658, 765 thousand 897, 567 thousand 435, 847 thousand 937, 485 thousand 395, 576 thousand 436, 783 thousand 785, 687 thousand 897.

56. Add 4 million 534 thousand 897, 3 million 778 thousand 697, 5 million 769 thousand 758, 6 million 836 thousand 745, 757 thousand 894, 367 thousand 376, 675 thousand 487, 456 thousand 737, 568 thousand 976, 79 thousand 754, 65 thousand 438, 79 thousand 768, 68 thousand 479, 82 thousand 397, 8 million 8 thousand 8.

57. Add 276 million 456 thousand 787, 956 million 876 thousand 545, 679 million 877 thousand 453, 466 million 788 thousand 567, 575 million 867 thousand 238, 698 million 458 thousand 487, 756 million 789 thousand 789, 469 million 756 thousand 574, 574 million 567 thousand 978, 885 million 745 thousand 765, 769 million 354 thousand 657, 433 million 778 thousand 844, 667 million 332 thousand 266, 589 million 746 thousand 457, 521 million 364 thousand 653.

58. Add 787 million 654 thousand 672, 545 million 678 thousand 659, 354 million 778 thousand 664, 832 million 768 thousand 575, 754 million 854 thousand 896, 987 million 987 thousand 657, 475 million 657 thousand 964, 765 million 887 thousand 976, 879 million 765 thousand 475, 567 million 547 thousand 588, 756 million 453 thousand 967, 448 million 877 thousand 334, 662 million 233 thousand 776, 754 million 647 thousand 985, 356 million 463 thousand 125.

	102.	*103.*	*104.*	*105.*	*106.*	*107.*	
59.	657	4689	75467	543876	7983648	6785468	*84.*
60.	453	7568	54874	756836	5867437	7584786	*85.*
61.	978	5674	43675	647479	4976877	8376854	*86.*
62.	867	6756	89763	876543	7387586	9367973	*87.*
63.	946	7568	54576	786436	8975438	4756856	*88.*
64.	587	8746	87658	475679	5467679	7678457	*89.*
65.	764	5978	48326	787536	4857748	4856738	
66.	648	8364	75897	569478	8485675	7948567	
67.	873	5799	89784	678797	5676487	8769858	*90.*
68.	764	8658	76348	956746	7834656	7989998	*91.*
69.	953	6784	57867	758667	5847589	8888888	*92.*
70.	564	7579	76845	876748	8795654	7575757	*93.*
71.	756	6874	54597	456685	7547878	9696699	*94.*
72.	965	7549	68745	874896	8567557	7685785	*95.*
73.	547	6793	85697	568437	5784995	8659897	
74.	976	5547	58746	875968	8659448	5784546	
75.	752	8796	76568	968475	7586876	8638756	
76.	587	5647	67875	697989	8979789	7987899	*96.*
77.	678	9876	58362	823424	9106758	8956785	*97.*
78.	756	7837	95764	756848	7647576	8678598	*98.*
79.	547	8768	69897	863685	5638975	6738957	*99.*
80.	679	5678	75645	697876	8756687	9763687	*100.*
81.	543	7847	68459	768756	6574835	8765868	*101.*
82.	689	8789	79876	897678	7689978	9776898	
83.	798	7648	86489	976578	4898767	8987976	

NOTE.—For examples 59 to 83 inclusive, read across the page.

For examples 84 to 89 inclusive, read vertically the successive columns within the limit of the brace; and so for examples 90 to 95 inclusive, and same manner for examples 96 to 101 inclusive.

For examples 102 to 107 inclusive, read vertically the entire column.

108. Add

	PROCESS.
29 dollars 59 cents 6 mills.	$29.596
48 dollars 27 cents 4 mills.	48.274
87 dollars 75 cents 7 mills.	87.757
98 dollars 87 cents 5 mills.	98.875
127 dollars 7 cents.	127.07
478 dollars 9 cents 6 mills.	478.096
	$869.668

	115.	*116.*	*117.*	*118.*	*119.*	*120.*
109.	$5.607	$36.089	$59.006	$68.02	$85.308	$165.
110.	7.81	45.004	67.507	74.326	97.009	276.06
111.	12.563	38.25	59.06	86.005	128.03	359.007
112.	27.	.978	.052	3.	.57	998.999
113.	139.875	48.764	57.372	49.078	584.27	875.
114.	237.	9.006	8.987	.06	263.	896.08

WRITTEN PROBLEMS.

121. Paid $367 for a horse, $2698 for a house, and $479 for furniture. How much did I pay for all?

122. A lot cost me $3964, a barn $996, fencing $1387, and farming tools $178. What did they all cost?

123. A ship sailed in one week 846 miles, the second week 958 miles, and the third week as much as the other two. How far did she sail in the three weeks?

124. Three vessels are loaded with lumber, as follows: the first carries 29368 feet, the second 1986 feet more than the first, and third 7568 feet more than the second. How many feet do they all carry?

125. Bought 4 car loads of wheat, paying for the first $878.36, for the second $1179.756, for third $1288.08, and for the fourth $1587.128. How much did I pay for the wheat?

126. Built a block of four houses; the two end ones cost me $4897.603 each, and the two middle ones $3579.875 each. What did the block cost me?

127. A and B each paid $29768.30 taxes, and C and D each $18679.796. How much did they all pay?

128. The water tax receipts during the first week of February, 1875, were $9367.50, the second week $8576.86, the third week $11975, and the fourth week $9987.87. What were the receipts for February?

129. Bought a house for $5897.48, a lot for $3673.62, and sold them both so as to gain $1976.50. How much did I receive for them?

130. Stocked four farms with sheep, putting into the first 5467, the second 4264, the third 876 more than the first, and the fourth 975 more than the second. How many sheep did I have?

131. A, B and C build a railroad, A receiving $12873.64 for his share, B $13560.92, and C as much as A and B and $589.78 more. How much did the railroad cost?

132. A real estate dealer sold 2 lots for $2976.40 each, and 2 others for $2745.80 each. How much did he receive for them all?

133. Bought 256 bushels of wheat for $378.75, 267 bushels of oats for $198.68, 176 bushels of corn for $119.96, and 398 bushels of hay seed for $1367.75. How much did they all cost? How many bushels were bought?

134. Bought a house for $4688, a lot for $3650; I sold the house at a gain of $627, and the lot at a gain of $565. How much did I receive for both?

135. A farmer raised in one year 698 bushels of corn, the second 987 bushels, the third 1289 bushels, the fourth 1565 bushels, and the fifth 1976 bushels. How much did he raise in the 5 years?

136. A flouring mill turned out in one week 1089 barrels of flour, in the second week 999 barrels, in the third 1467 barrels, and in the fourth 264 barrels more than in the second week. How many barrels were turned out in the 4 weeks?

137. Four drovers sent sheep to market, the first sending 976, the second 1079, the third 1285, and the fourth as many as the first and second. How many did they all send?

138. A, B, C and D, are farmers; A made in one year $1365, B $376 more than A, C $256 more than B, and D $468 more than C. How much did D make? How much did they all make?

139. A gentleman traveled in January 1687 miles, in February 987 miles, in March 1279 miles, and in April 279

miles farther than in February. How far did he travel in the 4 months?

140. A girl can count 5789 in one hour. How many can she count in 6 hours?

141. At a saw-mill there were cut in one week 16728 feet of lumber. How many feet can be cut in four weeks?

142. If one car contains 659 bushels of wheat, how many bushels in a train of 7 such cars?

143. Paid $5674.25 for one house. How much should I pay for 5 such houses?

144. Paid $3560 for a house, and $1880 for a lot. How much must I pay for 3 such houses, and 2 such lots?

145. A certain elevator will hold 6759 bushels of wheat, and 4678 bushels of corn. How many bushels of wheat and corn will 4 such elevators hold?

146. How many bricks may a mason put into 6 walls, if he can put 6798 bricks in one wall?

147. A railroad company carries 5375 passengers in one week. At the same rate how many passengers can they carry in 8 weeks?

148. A company earns $7987.60 during the month of January. At the same rate how much will they earn in the first four months of the year?

149. There are 5760 grains in one pound of silver. How many grains are there in 6 pounds of silver?

150. If it cost $12326 to build one mile of railroad, how much will it cost to build 6 miles?

151. If sound moves 1142 feet in one second of time, how far will it move in 10 seconds?

152. If a carrier pigeon can fly 987 miles in one day, how far can it fly in one week?

153. An army was composed of 9 regiments, and there were 976 men in each regiment. How many men in the army?

154. There are 365 days in one year. How many days in 20 years?

SECTION IV.

SUBTRACTION.

Art. 34. Since 6 units and 4 units are 10 units, 10 units are 4 units greater than 6 units, or 6 units are 4 units less than 10 units; also,

Since 12 units are 4 units more than 8 units, or 8 units are 4 units less than 12 units, 4 units is the difference of any two numbers one of which contains 4 units more or less than the other, hence

35. The ***Difference*** of two numbers is the number of units which one of the numbers is greater or less than the other.

Thus, 4 is the difference of 9 and 5.

36. ***Subtraction*** is the process of obtaining the difference of two numbers.

37. The terms employed in subtraction are *minuend*, *subtrahend* and *difference*.

38. The ***Minuend*** is the larger of the two numbers whose difference is required, and is the number to be diminished by the operation of subtraction.

Thus, 7−3=4, 7 is the minuend.

39. The ***Subtrahend*** is the less of the two numbers whose difference is required, and is the number to be subtracted from the minuend.

Thus, 7−3=4, 3 is the subtrahend.

40. The ***Sign*** of ***Subtraction*** is −, called *minus*. When placed between two numbers it denotes that the

number on the right of the sign is to be subtracted from the one on the left.

Thus, 12−9=3.

NOTE.—The difference is sometimes called the *remainder*, especially when a part of a number is to be subtracted from the whole number; as, 5 yards of muslin are cut from a piece containing 20 yards.

PRINCIPLE.

The minuend and subtrahend are always *like numbers.*

ORAL PROBLEMS INVOLVING BUT ONE PROCESS.

1. A boy had 20c., and paid 15c. for a slate. How much had he left?

ANALYSIS.—Since he had 20c., and paid 15c. for a slate, he had left as many cents as 20c. is greater than 15c., which are 5c. Therefore he had 5c. left.

2. Paid 14c. for paper, and 9c. for a pencil. How much more was paid for the paper than for the pencil?

3. Mary is 16 years old, and her brother is 8 years old. What is the difference of their ages?

4. John bought 18 marbles, and James 12. How many more did John buy than James?

5. A lady having $19, paid $8 for a shawl. How much did she have left?

6. A boy gathered 21 quarts of nuts, and sold 12 quarts. How many quarts were left?

7. A man bought 28c. worth of meat, and gave the trader 35c. How much change should he receive?

8. Bought 38c. worth of cotton cloth, and gave the merchant 50c. How much change should I receive?

9. Bought a suit of clothes for $36, and gave the dealer $50. How much change should I receive?

10. There are two trees in my yard, one of which is 48 feet high, and the other is 13 feet less. How high is the other tree?

11. A gentleman is 60 years old, and his wife is 12 years younger. How old is the wife?

12. A farmer raised 75 bushels of corn this year, which is 15 bushels more than he raised last year. How much did he raise last year?

13. Paid $90 for a wagon, and sold it so that I lost $20. How much did I receive for the wagon?

14. Took 80 bushels of oats to market, which is 30 bushels more than I left at home. How many bushels did I leave at home?

15. Sold 100 bushels of wheat, which was 30 bushels more than I kept. How many bushels did I keep?

16. Bought 88c. worth of groceries, and gave the grocer $1. How much change should I receive?

ORAL PROBLEMS INVOLVING TWO PROCESSES.

17. Paid 7c. for a pencil, 12c. for a book, and gave 25c. to the dealer. How much change should I receive?

18. A lady had $12, found $10, and afterward lost $8. How much did she have left?

19. Had 50c., paid 30c. for a book, and 12c. for toys. How much had I left?

20. A farmer kept 45 hogs in 3 pens; in one pen were 20 hogs, in the second 15. How many were there in the third?

21. Mary had 12 peaches, Jane 9 peaches, and Sarah 30 peaches. How many peaches has Sarah more than both Mary and Jane?

22. A wagon cost $50, a sleigh $20, and harness $10. How much does the wagon cost more than both sleigh and harness?

23. Bought 40c. worth of sugar, and 30c. worth of coffee. How much change should I receive if I gave the grocer $1.

24. A hogshead contains 63 gallons of water. If I pour

into it at one time 40 gallons, at another 14 gallons, how many gallons more will fill it?

25. A tree is 75 feet high. If 40 feet be broken off at one time, 20 feet at another, how much will yet remain?

26. A boy put 18 chickens into one coop, 12 into another, and enough in the third to make 45 altogether. How many did he put into the third coop?

27. A woman paid 50c. for some tea, 25c. for cheese, and enough for oil so that they all cost $1. How much did she pay for oil?

28. A farmer planted an orchard of 100 apple trees in 4 days. The first day he planted 30 trees, the second 30 trees, the third 20 trees. How many did he plant the fourth day?

29. A hunter in 4 days shot 80 birds. The first day he shot 24 birds, the second 24 birds, and the third 12 birds. How many did he shoot the fourth day?

30. A gentleman lost $20 each day for 3 days, and enough the fourth day to make his whole loss $80. How much did he lose on the fourth day?

31. From a pile of wheat containing 72 bushels, 20 bushels were taken at one time, 30 bushels at another, and 10 bushels at another. How many bushels were left?

32. Gave 16c. for a pound of sugar, 20c. for a pound of coffee, and 45c. for meat. How much more was given for meat than for both sugar and coffee?

33. Gave 20c. for oranges, 30c. for lemons, and 41c. for candies. How much less did I pay for candies than for the lemons and oranges?

34. Sold a pencil for 9c., a slate for 12c., and some toys for 8c. If I received a 25c. piece, and a 10c. piece, how much change should I give?

35. Bought a coat for $25, a pair of pants for $10, and a vest for $6. If I give a $50 bill, how much should I receive in change?

36. A lady put up at one time 22 quarts of cherries, at another 10 quarts, and at another 8 quarts. She lost at one

time 12 quarts, at another 9 quarts. How many quarts were left?

37. A boy had 9 marbles, found 3 more than he had. He afterward lost 6 less than he found. How many marbles did he have left?

38. A squirrel ran up a tree 20 feet, then down 8 feet, up 4 feet, down 12 feet, up 9 feet, up 7 feet, down 11 feet, up 7 feet, down 6 feet, down 9 feet, up 25 feet, down 10 feet, down 9 feet, up 12 feet, down 16 feet. How high is the squirrel?

ORAL COMBINATIONS.

39. $15+7+8+9-7-6-4+8-5+9-7+6+9=?$

40. $12+9-7+5+9-10+6-12+7+5+3-12+5=?$

41. $7+8+6-9-3+6+11-10-8+5+4-6-5=?$

42. $50-8-7-6-5+7-5+4-5+8-9+6-8+12=?$

43. $18+12-8-8-8+12+5+4-10-7+5+3-9=?$

44. $16+8-9+15-9-9+6-9+5-7+12+5-20=?$

45. $24+8+8-20+7+9-8+6-4+9+6-5-10=?$

46. $9+6-8+7+8+8-6-6+12+10+10-20=?$

47. $12+5+5+5-7-7+6-5+9+9-8-6+8+6=?$

48. $18-6-4-8+11+5-10+20+8+7+9-40=?$

49. $12+5+4+3-5-5+9+8+9+8-5-5-5-5=?$

50. $7+8+9-3+9-3+9-3+9-3+9+3-9-2=?$

51. $17+7-12+7+6-5+9-4+8+7-8+6+6=?$

52. $16+12+12-10-10+8-9+7-9+6-9+12=?$

53. $8+9+6+6-11-9+12+5+4-10-9-8+6=?$

54. $11+9-6+11-5+20+20-50+21+9-8-12=?$

55. $60-4-4-4-4-4-4-4-4-4+8+8-7-7=?$

56. $40+40+20+60-100-40+80-40-50+90=?$

57. $12+12+12+12+12+12-2-10-20-30+90=?$

58. $11+11+11+11+11+11-6+12+3-5-5-5=?$

59. $25+10+5-8-7+11-12+6-20+19+6+7=?$

60. $30+20-40+15+25+50+100+200+400+800=?$

61. $500-300-100-50+25+25-30-30-30-10=?$

62. $400-200-200+80+20+100+1000-200-500=?$

63. $1000-400-300-200+300+400+200-1000=?$

64. A boy had 144 chestnuts. He then took 7 away each day until there was left a number less than five. How many were left?

65. If there are 133 apples in a barrel, and George should take out 8 each day, and James 9 every other day, how many apples will George get if he precedes James?

66. A man has 63 volumes of books in his library. 2 shelves contain 9 volumes each of poetry; 1 shelf Dickens' works in 13 volumes, and Hood's works in 5 volumes; another shelf Barnes' Notes in 11 volumes; the remainder consists of 3 Bibles and the "Little Classics." How many volumes of "Little Classics" in the library?

ANALYSIS OF SUBTRACTION.

67. From 9876 subtract 3452.

PROCESS.

9876
3452
——
6424

ANALYSIS.—1. Write the numbers so that figures of the same order shall stand in the same column.

2. Begin at the order of units thus: 6 units less 2 units are 4 units. Write the 4 units beneath the column of units.

3. 7 tens less 5 tens are 2 tens. Write the 2 tens beneath the column of tens.

4. 8 hundreds less 4 hundreds are 4 hundreds. Write the 4 hundreds beneath the column of hundreds.

5. 9 thousands less 3 thousands are 6 thousands. Write the 6 thousands beneath the column of thousands.

Therefore, 9876−3452=6424.

68. From 8345 subtract 4879.

PROCESS.

8345
4879
——
3466

ANALYSIS.—1. Write the numbers as above.

2. 5 units of the minuend are less than 9 units of the subtrahend. To supply this deficiency, take 1 ten of the 4 tens of the minuend, and change it to units. 1 ten is equal to 10 units, which, added to the 5 units is equal to 15 units. 15 units less 9 units are 6 units. Write the 6 units in the order of units.

3. 1 ten having been taken from the 4 tens of the minuend, 3 tens are left. 3 tens of the minuend is less than 7 tens of the subtrahend.

To supply the deficiency, take 1 hundred of the 3 hundreds of the minuend and change it to tens. 1 hundred is equal to 10 tens, which, added to the 3 tens is equal to 13 tens. 13 tens less 7 tens are 6 tens. Write the 6 tens in the order of tens.

4. 1 hundred having been taken from the 3 hundreds of the minuend, 2 hundreds are left. 2 hundreds of the minuend is less than 8 hundreds of the subtrahend. To supply the deficiency, take 1 thousand of the 8 thousands of the minuend, and change it to hundreds. 1 thousand is equal to 10 hundreds, which, added to 2 hundreds, is equal to 12 hundreds. 12 hundreds less 8 hundreds are 4 hundreds. Write the 4 hundreds in the order of hundreds.

5. 1 thousand having been taken from the 8 thousands of the minuend, 7 thousands are left. 7 thousands less 4 thousands are 3 thousands. Write the 3 thousands in the order of thousands.

Therefore, 8345−4879=3466.

69. From 2000 subtract 764.

PROCESS.

2000
764
1236

ANALYSIS.—1. Write the numbers as above.

2. 2000=1 thousand 9 hundreds+9 tens+1 ten. 1 ten=10 units. 10 units less 4 units are 6 units. 9 tens less 6 tens are 3 tens. 9 hundreds less 7 hundreds are 2 hundreds. 1 thousand less zero is 1 thousand.

Therefore, 2000−764=1236.

PROOF OF SUBTRACTION.

The Subtrahend plus the Difference is equal to the Minuend.

NOTE.—The following examples will serve to illustrate the principles of subtraction:

1. The minuend is 8469; the subtrahend is 7883. What is the difference?

2. The subtrahend is 7546; the difference is 2786. What is the minuend?

3. The minuend is 15342 acres; the difference is 5679 acres. What is the subtrahend?

4. The subtrahend is $9457; the difference is $7638. What is the minuend?

5. The difference is 12758 feet; the subtrahend 25697 feet. What is the minuend?

WRITTEN EXERCISES.

NOTE.—The following table may be used in the following manner, for instance: Find difference of ex. 10, 2d col., and ex. 9, 3d col.
Ex. 10, 2d col., is 9067, and ex. 9, 3d col., 65048.

1st col.	2d col.	3d col.	4th col.	5th col.	6th col.
1.—567	3058	87054	567438	7643878	5683876
2.—485	2360	74508	875643	5678545	6838765
3.—369	4789	45087	756798	6754856	8378659
4.—484	3076	60379	797853	7356871	3786598
5.—673	4508	73097	368974	6785713	6597384
6.—767	5804	84650	486743	4743658	9738465
7.—981	8054	46085	675858	3657484	7384659
8.—867	6079	58064	864307	9764583	3847396
9.—753	7906	65048	705498	7489653	4659738
10.—862	9067	79006	890649	4957862	8465973
11.—374	5432	60079	347681	8956748	6000005
12.—219	4352	90607	475683	7862495	7600005
13.—472	2435	58636	643895	9578624	9856703
14.—865	7858	83568	436598	5786249	5607398
15.—974	5897	63865	698543	8624957	6709396
16.—749	9785	97091	756784	5648947	9385607
17.—497	7589	71984	465778	7564894	8000003
18.—673	5647	49178	875647	4756489	5993601
19.—763	6475	87987	587476	8946754	9000075
20.—850	4576	27964	835658	6489475	8000004

WRITTEN PROBLEMS INVOLVING BUT ONE PROCESS.

70. Paid \$5431 for a house, and \$2895 for a lot. How much did the house cost more than the lot?

PROCESS.

\$5431
2895
\$2536

ANALYSIS.—Since the house cost \$5431, and the lot \$2895, the house cost as many dollars more than the lot as \$5431 is greater than \$2895, which is \$2536.

71. Bought a cargo of wheat for \$4526, and a cargo of corn for \$2768. How much did the corn cost less than the wheat?

PROCESS.

$$\begin{array}{r} \$4526 \\ 2768 \\ \hline \$1758 \end{array}$$

ANALYSIS.—Since the wheat cost $4526, and the corn $2768, the corn cost as many dollars less than the wheat as $2768 is less than $4526, which is $1758.

NOTE. — The teacher will carefully impress upon the minds of the pupils the distinction of these Analyses.

72. A mason put into the east wall of a building 1234 bricks, and into the south wall 987 bricks. How many more bricks in the east wall than in the south wall?

73. The population of a certain town in 1870 was 3467; and in 1874, 2399. How much less was the population in 1874 than in 1870?

74. A gentleman's income one year was $2958, and the next year it was $4120. How much greater was his income the second year than the first?

75. Two vessels start from the same point and travel in the same direction; the one 7821 miles, the other 5078 miles. How far apart are they?

76. A gentleman owns two vessels, one of which will carry 25763 feet of lumber, and the other 20976 feet. How much will the first vessel carry more than the second?

77. A farmer dug two trenches, one of which will contain 11000 gallons of water, the other 8975 gallons. How many gallons does the latter contain less than the former?

78. In the Chicago Water Works one of the engines will pump 38000000 gallons per day, another 18675450 gallons. How many gallons more does the larger engine pump than the smaller, per day?

79. Gave $15234 for a farm, and $8796 less for the buildings upon it. How much did the buildings cost?

80. A gentleman having $55381, expended $16897 for cattle. How much had he left?

81. A man having an estate worth $78346, bequeathed to his two sons $49758. How much did he retain?

82. A man bought a quantity of land for $12975, and sold it for $15250. How much did he gain?

83. Illinois contains 55405 square miles, and Indiana 33809. How much larger is Illinois than Indiana?

84. Ohio contains 39964 square miles, and Oregon 100000. How much less is Ohio than Oregon?

85. The State of Ohio in 1870 raised 20539643 pounds of wool, and California 11391743 pounds. How much more was raised in Ohio than in California?

86. The same year New York produced 167147526 pounds of butter, and Pennsylvania 60834644 pounds. How much is the difference of their production?

87. The population of Chicago in 1870 was 298977, and that of St. Louis was 310864. Which was the larger, and how much?

88. In 1870 the population of a certain city was 71440; it has increased so that the population now, 1876, is 83578. How much has been the increase?

89. In 1775 the war of the Revolution began, and in 1861 began the Civil War of the U. S. How many years apart were they?

90. The highest mountain in Asia is 29100 feet, and the highest in the United States is 17900 feet. What is the difference of their heights?

91. New York in 1870 raised 652800 horses, and Illinois 1008800. Which state produced the most horses, and how many?

92. The taxes of a certain state were, in 1874, $9520100, and the following year $789890 less. How much were they in the following year?

93. The receipts of a certain city for one year were $9223012, and the expenditures $8845764. How much were the receipts greater than the expenditures?

WRITTEN PROBLEMS INVOLVING TWO OR MORE PROCESSES.

94. A gentleman bought two farms; for one he paid $5476, and for the other $4562. He sold them both for $12580. How much did he gain?

95. Having \$12847, I sold a house for \$5489; how much more do I need so that I shall have \$23425?

96. A man worth \$15300, gave \$4360 to each of his two sons, and \$4225 to his daughter, and the remainder to charitable purposes. How much did he give in charity?

97. Having \$5895, I lost in trade \$2750, then made \$3560, afterward lost \$4680, and then made \$3690. How much did I have then?

98. From 11 million 300 thousand take the difference of ten thousand seventy and one hundred four.

99. From 5 million 71 thousand five, take the difference of 2 hundred thousand 17, and 61 thousand 7, increased by their sum.

100. Vice-President Wilson died in 1875 at the age of 63. The Declaration of Independence occurred in 1776. How many years before the birth of Henry Wilson was Independence declared?

101. A gentleman owning 589 acres of land, bought 349 more; he afterward sold at one time 294 acres, and at another 57 acres. How many acres were left?

102. If I travel Monday 2756 miles from home, Tuesday 1907 miles towards home, Wednesday 3874 miles from home, and Thursday return 2009 miles; how far from home am I Thursday night?

103. A brickmaker had 2 kilns, each containing 43567 bricks; he sold 8376 at three different times, and used 12564. How many bricks had he left?

104. London, the largest city in the world, had in 1870 a population of 3082300, and New York, the largest city in America, a population of 942292. If, in a certain time, New York has an increase of 17058, and London 39856, how many more inhabitants will London have than New York?

105. From a bin containing 80071 bushels of salt, a merchant sold to one man 1020 bushels, to another 129 bushels, and to another the remainder. How many bushels did the last man buy?

106. An apple dealer in Chicago finds that out of 1849 +956+987 barrels of apples sent to him from Michigan, 280 barrels of them are damaged. How many orders of 800 bbls. can be filled from the remainder? How many barrels will be left?

107. Bought a house, lot, horse, and chaise, for $17800. If I paid $7890 for the lot, and $480 for the horse and chaise, how much was paid for the house?

108. If 11500500 and 12567658 be added to 7984307, the sum will be the number of inhabitants in France in 1830. The population of the British Empire at the same time was 22297621. Which country had the larger population? and how many more?

109. A speculator bought 3 houses, paying for the first $7010, for the second $13000, and for the third as much as for the first and second, less the number of dollars which the second cost more than the first. How much did he pay for the third house?

110. The source of the Missouri River is 6800 feet above the level of the sea; that of the Mississippi is 1680 feet above the sea level. A certain spring is located 1009 feet below the source of the Missouri. How far above or below the source of the Mississippi is it?

111. 3 houses are found to be standing in a straight line; the second 5420 feet from the first, and the third 7337 feet from the second. A certain tree standing between the second and third is 538 feet from the third. How far is the tree from the first house?

112. $6385 was paid for each of four houses, which were soon after sold so as to lose $987 on each of two of them. How much was received for the houses?

113. A commission merchant received 7598 barrels of flour for each of six months, and sold 7438 for each of five months. How many barrels has he left?

114. If I can save $568 a year from my income, how much less than $3600 can I save in 6 years?

115. Bought a house, farm, and 4 horses for $15310. If I paid $4375 for the house, and $560 for the horses, how much does my farm cost?

116. A herds 19020 sheep, B 12780, and C 5460 less than A. How many sheep more or less does C herd than B?

117. A, B and C, form a partnership with a capital of $25800; if their gains the first 3 years are respectively $1800, $2500 and $4875, and their losses $980, $650 and $298, what are they all worth at the end of that time?

118. An army of 100000 men engaged in battle, and was re-enforced by 29000 more; 237 deserted, 12888 were killed, 1500 were missing. How many were left in the army?

119. The population of New York is about 4362834, of New Jersey 905794, of Pennsylvania 3519601, and of Delaware 125015. How much is the population of New York greater or less than the three other Middle States?

120. In the year 1860 the number of teachers employed in the public schools of Illinois was 11099, and the number of pupils in attendance 433018; in Massachusetts the number of teachers 5308, and of pupils 206974; in New York the number of teachers 15872, and of pupils 697273. How many more pupils than teachers in these States?

121. Three brothers bought a factory, the first paying $5682.40, the second $8624.80, and the third as many dollars more than the second as the second paid more than the first. How much did the third brother pay?

122. The brothers mentioned in the above example sold the factory for $30450, the first receiving $932.50 more than he paid, the second $1720.60 more than he paid, and the third the remainder. How much did the third brother receive?

123. A man purchased a house and farm for $7630.50; he expended $1230 in repairing the house, $987.50 in improving the farm, and $556.75 in planting an orchard, when he sold both for $12579. How much did he gain?

124. Bought a quantity of grain of one man for \$1210.60, another quantity of another man for \$929.85, a third quantity of a third man for as many dollars less than the second as the second is less than the first. How much was paid to the third man?

125. If a man's income is \$200 a month, and his expenses are \$35 a month for rent, \$48.60 for grocery bills, and \$45.75 for other expenses, how much can he save in 6 months?

126. The difference of \$17846 and \$27984, is what was paid for a certain property, it was soon after sold for \$14025. How much was the gain?

127. George Washington died in 1799 at the age of 67 years. How long before his birth was the discovery of America in 1492?

128. C has \$15890; A has \$1546 less than C. How many dollars has A? B has \$6430 less than A; how many dollars has B? D has \$62 more than A and B together; how many dollars has D?

129. A farmer filled 4 bins with grain. Into the third he put 2240 bushels; into the first, 420 bushels less than into the third; into the second, 840 bushels less than into the first; and into the fourth he put 650 less than into the first and second together. How many bushels did he put into each bin?

130. A farmer being asked how many sheep he had, replied that if he had 420+560 more, he should have 3628 sheep. How many had he?

131. After A, B, C, and D had traded for a year, it was found that C owned \$21360, that A owned \$930 less than C, that B owned \$930 less than A, and that D owned \$930 more than B. How many dollars did each own?

132. A gentleman dying bequeathed his property, consisting of real estate to the value of \$380500, and bank stocks amounting to \$360500, as follows: to his brother \$95640; to his sister as much as to his brother, less \$15000;

to his son \$59880 more than to his sister; to his wife as much as to the other three plus \$18000; and the remainder to a charity. How much was given to charity?

133. A grain merchant in Chicago received two orders from New York, one for 15000, the other for 10000 bushels of corn, and a smaller order. In fulfillment of these orders he shipped 31780 bushels. How much was the smaller order?

134. From three times the sum of eighty-seven thousand six hundred four, forty-nine thousand three hundred fifty-eight, and twenty-nine thousand one hundred ninety, take four times the difference of the first two numbers, and tell how much the remainder exceeds the last of the given numbers?

135. Bought 5 houses, each for \$6788, and gave in payment 6 lots, each for \$3790, and the rest in money. How much money did I give?

136. A revenue cutter cruising in the Pacific Ocean started from a certain point and sailed north 1525 miles, then south 825 miles, then north 728 miles, then south 960 miles, then north 588 miles, then south 1438 miles, then north 980 miles, then south 840 miles. How far was the vessel from the starting point? In what direction?

137. An aeronaut ascended from a certain point 3987 feet during the first hour, descended 1876 feet the second hour, ascended 2019 feet the third hour, dropped down 2569 feet the fourth hour, rose 3289 feet the fifth hour, sailed due east 12 miles the sixth hour, and then descended 3429 feet the seventh hour. How far from the earth is the aeronaut at the end of the seventh hour?

SECTION V.

MULTIPLICATION.

Art. 41. A farmer gave to each of his 4 sons 5 acres of ground. How much did he give them all?

PROCESS OF ADDITION.

5 acres + 5 acres + 5 acres + 5 acres = 20 acres.

ANALYSIS.—Since the farmer gave to each son 5 acres, he gave to the 4 sons the sum of 5 acres + 5 acres + 5 acres + 5 acres = 20 acres.

The above is an illustration of examples in which it is required to obtain the sum of several numbers each equal to the other, and the method of obtaining this sum may be either by the above process called "process by addition," or by a much shorter process, as follows:

PROCESS BY MULTIPLICATION.

4 times 5 acres = 20 acres.

ANALYSIS.—Since the farmer gave to each son 5 acres, he gave to the four sons 4 times 5 acres = 20 acres.

This latter method is called the "process by multiplication." Hence,

42. ***Multiplication*** is a short method of obtaining the sum of several equal numbers.

Thus, instead of saying 3 + 3 + 3 + 3 + 3 = 15, we say 5 times 3 = 15, or 5 3's = 15. When the numbers to be added are both large and numerous, the process by addition would be very laborious.

43. The terms employed in multiplication are *multiplicand*, *multiplier*, and *product*.

44. The ***Multiplicand*** is any one of the equal numbers to be added.

Thus, in the problem above, 5 acres is the multiplicand.

45. The ***Multiplier*** is the number of equal numbers to be added.

Thus, in the same problem, 4 is the multiplier.

46. The ***Product*** is the sum of the equal numbers to be added.

Thus, in the same problem, 20 acres is the product.

47. The ***Sign*** of ***Multiplication*** is $\times$, called *times* or *multiplied by*. When placed between two numbers it indicates that one of the numbers is to be multiplied by the other.

Thus, $3\times5=15$, is read 3 times 5 is 15, or 5 times 3 is 15; also, $\$4\times5=\$20$, is read 5 times \$4 is \$20, or \$4 multiplied by 5 is \$20.

48. The multiplicand and multiplier are called ***Factors*** of the product.

Thus, 3 and 5 are factors of 15.

PRINCIPLES.

1. The multiplicand may be either an abstract or denominate number.
2. The multiplier is always an abstract number.
3. The product is always like the multiplicand.
4. The multiplication or division of either factor by any number, multiplies or divides the product by the same number.

NOTE. — When both factors are abstract numbers, either may be used as the multiplicand or multiplier, but if one of these numbers is denominate, it must be considered the multiplicand.

ORAL PROBLEMS.

1. There are 4 quarts in 1 gallon. How many quarts are there in 5 gallons?

GENERAL STATEMENT.—In 5 gallons there are 5 times as many quarts as there are in 1 gallon.

ANALYSIS —Since there are 4 quarts in 1 gallon, in 5 gallons there are 5 times 4 quarts, which are 20 quarts.

Therefore, in 5 gallons there are 20 quarts.

NOTE. — It will be found a valuable exercise to require pupils to make a general statement of the solution of the problem.

2. What will be the cost of 6 oranges, if one orange cost 5c.?

3. If one ton of coal cost $7, how much will 5 tons cost?

4. There are 8 quarts in one peck. How many quarts are there in 4 pecks?

5. How many pecks are there in 9 bushels, if there are 4 pecks in 1 bushel?

6. How much can a man earn in 6 days, if he can earn $8 in one day?

7. A gentleman gave $12 to each of his 5 sons. How much did they receive?

8. A person traveled 12 miles a day for 7 days. What distance did he travel?

9. A boy can earn $9 a month. How much can he earn in 9 months?

10. There are 12 inches in one foot. How many inches in a pole 8 feet long?

11. There are 3 feet in one yard. How many feet are there in 20 yards?

12. How many inches are there in 2 yards?

13. There are 2 pints in one quart. How many pints are there in 5 gallons?

14. Bought 12 yards of broadcloth at $12 a yard. How much was the cost?

15. Gave $6 a head for sheep, and $12 a head for hogs. How much will 5 sheep and 5 hogs cost?

16. What is the cost of 12 pairs of boots at $8 a pair?

17. Bought 9 yards of cloth at $5 a yard. How much change should I receive if I offer the trader $50.

18. A lady had a $50 note, a $20 note, and a $5 note. How much money had she left after buying 6 sheep at $12 each?

19. There are 16 ounces in one pound of tea. How many ounces in 5 pounds?

20. Bought 6 oranges at 5c. each, and 5 lemons at 8c. each. How much did they all cost?

21. Bought 12 yards of cloth, and 5 yards of velvet; the cloth at $5 a yard, and the velvet at $8 a yard. How much did I pay for both?

22. How many days are there in 9 weeks? In 12 weeks? In 20 weeks?

23. Built two fences; one 45 feet long, the other 12 yards long. Which is the longer, and how much?

24. There are 9 square feet in one square yard. How many square feet in a floor that contains 12 square yards?

25. There are 4 quarts in one gallon. What is the cost of 3 gallons of oil at 12c. a quart?

26. There are 3 feet in one yard. What is the cost of 12 yards of carpet at $1 a foot?

27. Since there are 4 pecks in one bushel, what is the cost of 5 bushels of peaches, at $2 a peck?

28. Bought 5 tons of coal at $12 a ton, and 6 cords of wood at $5 a cord. How much was paid for both coal and wood?

29. What is the cost of one gallon of molasses at 20c. a quart?

30. Paid 75c. for a bushel of potatoes, and sold them at 20c. a peck. How much did I gain or lose?

31. If a boy can earn $2 a day, how much can he earn in 6 weeks, omitting Sundays?

32. A man paid $6 for a vest, twice as much for a pair of pants as for the vest, and twice as much for a coat as for the pair of pants. How much did he pay for the suit?

33. Bought 3 boxes of raisins at $5 a box, and 5 boxes of lemons at $3 a box. How much did they both cost?

34. Mary reads twice each day. How many times will she read in 4 weeks, of 5 days each?

35. John counted 7 nines, and Mary 6 twelves. How many more did Mary count than John?

36. In a cornfield there are 8 rows of corn, having 9 hills in each row; in another there are 7 rows, having 7 hills in each row. If the crows tear up 2 hills in the first field, and 3 rows in the second field, how many hills will remain? How many hills should be replanted?

37. There are 8 furlongs in one mile. How many furlongs are there in 9 miles and 6 furlongs?

38. Since there are 12 inches in one foot, how many inches are there in 12 feet and 6 inches?

39. A squirrel carried into his nest 5 acorns each day for 2 weeks. How many acorns did he gather?

40. A farmer sold 5 tons of hay at $12 a ton, and took in exchange 7 barrels of flour at $8 a barrel, and the remainder in cash. How much cash did he receive?

41. One farmer took to market 8 turkeys, weighing 12 pounds each, and another took 12 geese, weighing 9 pounds each. How much more or less did the turkeys weigh than the geese?

42. Bought 12 yards of ribbon, at 12c. a yard, and gave in payment 120c. How much do I still owe?

43. Two boys start from the same place and travel in the same direction, one at the rate of 5 miles an hour, the other at the rate of 9 miles an hour. How far apart will they be in 12 hours?

44. Two men start from the same place and walk in opposite directions, one at the rate of 4 miles, the other at the rate of 5 miles an hour. How far apart will they be in 12 hours?

45. A farmer started to market with 8 bushels and 3 pecks of wheat, and sold 5 bushels and 2 pecks to one man, and 3 pecks to another. How much wheat had he left?

46. Bought 9 tons of coal at $12 a ton, and gave in payment 12 barrels of flour at $8 a barrel, and the rest in cash. How much cash did I give?

47. Sold 12 yards of cloth at $7 a yard, receiving in payment 9 head of sheep at $8 a head, and the remainder in money. How much money did I receive?

48. Traveled on a journey afoot for 6 days at 12 miles a day, and returning rode 2 days at 25 miles per day, and walked the remaining distance home. How far did I walk on my return? How many miles did I walk on the journey?

49. I have a box divided into 2 parts; in each part there are 3 parcels; in each parcel there are 4 bags; in each bag there are 5 marbles. How many marbles are there in the box?

50. There are 4 farmers, each of whom has 5 fields of pasture; each field has 4 corners, and in each corner there are 9 sheep. How many sheep do the farmers own?

51. Since there are 4 pecks in one bushel, how many pecks are there in 9 bushels and 3 pecks? in 12 bushels and 2 pecks?

52. Since there are 12 inches in one foot, how many inches are there in 6 feet and 6 inches? In 9 feet 7 inches? In 12 feet 8 inches?

53. Since there are 8 furlongs in one mile, how many furlongs are there in 7 miles and 5 furlongs? In 9 miles and 6 furlongs? In 12 miles and 7 furlongs?

54. A lady bought a gold chain weighing 2 pwt. 12 grains. How many grains did it weigh, if there are 24 grains in one pwt.

55. Bought 5 ounces 15 pennyweights of gold, at $1 per pennyweight. How much did it cost if there are 20 pennyweights in one ounce?

56. Sold 8 pounds 4 ounces of drugs at 9c. an ounce. How much was received for it if there are 12 ounces in one pound?

57. Find cost of 12 gallons 2 quarts of oil at 10c. a quart; there being 4 quarts in one gallon.

58. At 8 cents each, how much will 4 dozen and 2 lead pencils cost?

59. A gentleman receives a salary of $72 a month, and his son a salary of $60 a month. How much more does the father receive than the son in 9 months?

60. Bought 5 pounds 10 ounces of wool at 4c. an ounce. What is the cost if there are 16 ounces in one pound?

ANALYSIS OF MULTIPLICATION.

61. What is the product of 6 times 72?

PROCESS BY ADDITION.	PROCESS BY MULTIPLICATION.
72	72
72	6
72	—
72	432
72	
72	
—	
432	

ANALYSIS.—The analysis by addition has been explained already, Art. 41. The analysis by multiplication is as follows:

1. For convenience write the multiplier beneath the multiplicand.

2. Multiply each figure of the multiplicand by the multiplier, as follows: 6 times 2 units are 12 units, equal to 1 ten and 2 units. Write the 2 units in the order of units, and add the 1 ten to product of 6 times 7 tens, or 42 tens. 42 tens plus 1 ten are 43 tens, equal to 4 hundreds and 3 tens. Write the 3 tens in the order of tens, and the 4 hundreds in the order of hundreds.

Therefore, the product of 6 times 72 is 432.

62. What is the product of 46 times 3467?

PROCESS BY MULTIPLICATION.

3467 Multiplicand.
46 Multiplier.

20802 Partial product, 3467×6.
13868 Partial product, 3467×40.

159482 Entire product, 3467×46.

ANALYSIS.—1. Write the multiplier as before, beneath the multiplicand, and multiply each figure of the multiplicand by each figure of the multiplier, beginning at the order of units; thus, 6 times 7 units are 42 units, equal to 4 tens and 2 units. Write the 2 units in the order of units, and add the 4 tens to the product of 6 times 6 tens.

2. 6 times 6 tens are 36 tens. 36 tens plus 4 tens are 40 tens, equal to 4 hundreds and 0 tens. Write the 0 tens in the order of tens, and add the 4 hundreds to the product of 6 times 4 hundreds.

3. 6 times 4 hundreds are 24 hundreds. 24 hundreds plus 4 hundreds are 28 hundreds, equal to 2 thousands and 8 hundreds. Write the 8 hundreds in the order of hundreds, and add the 2 thousands to the product of 6 times 2 thousands.

4. 6 times 3 thousands are 18 thousands. 18 thousands plus 2 thousands are 20 thousands, equal to 2 ten-thousands and 0 thousands. Write the 0 thousands and the 2 ten-thousands in their respective orders.

Therefore, 20802 the product of 3467×6 is a partial product of 3467×46.

5. The second partial product is obtained by multiplying 3467 by the 4 tens or 40 units; thus, 40 times 7 units are 280 units, equal to 2 hundreds and 8 tens. (The 0 units may be rejected.) Write the 8 tens in the order of tens, and add the 2 hundreds to the product of 40 times 6 tens. Proceeding as before, the second partial product, 138680, is obtained.

Adding the partial products, the entire product is found to be 159482.

Therefore, the product of 46 times 3467 is 159482.

The same result is obtained by observing the following directions: Multiply each figure of the multiplicand by each figure of the multiplier, writing the first figure of each partial product in the order occupied by that figure of the multiplier producing the partial product.

PROOF OF MULTIPLICATION.

I. Subtract the multiplicand from the product, and then from the remainder, and so continue until the number of subtractions equals the number of units in the multiplier.

II. Division. Article 61.

The continued product of several numbers is indicated by placing the sign $\times$ between each two of the numbers.

Thus, $3 \times 4 \times 5 \times 6 = 360$, indicates the continued product of 3, 4, 5, 6, which is obtained by multiplying one of the numbers by another, and the product by a third number, etc.

NOTE.—If, as sometimes happens, the multiplier is larger than the multiplicand, the latter, for convenience in practice, may be used as the multiplier.

WRITTEN EXERCISES.

63. Multiply the following numbers by each of the numbers from 2 to 12 inclusive.

NOTE.—These exercises may serve to promote rapidity of execution.

1.—58745	13.—900195	25.—1967341	37.—20907683
2.—63294	14.—354764	26.—4192093	38.—42765401
3.—82563	15.—822073	27.—8765437	39.—22663973
4.—42937	16.—323599	28.—9988776	40.—19977991
5.—54012	17.—765102	29.—4039007	41.—83215946
6.—89645	18.—358455	30.—2595139	42.—18671868
7.—54785	19.—839768	31.—9611437	43.—73689202
8.—49236	20.—467453	32.—3902914	44.—12345678
9.—36528	21.—370228	33.—7856374	45.—91223344
10.—73924	22.—995323	34.—6778899	46.—79911997
11.—21045	23.—201567	35.—7129304	47.—64951238
12.—54698	24.—554853	36.—9315925	48.—89012345

Any integral number may be multiplied by 10, by 100, by 1000, etc., by annexing to the multiplicand as many ciphers as there are ciphers in the multiplier.

Thus $34 \times 100 = 3400$.

64. Find the products of the following factors by multiplying the multiplicand by the significant figures of the multiplier, and annexing to the product thus obtained as many ciphers as there are on the right of the multiplier.

1.—425×900	6.—5468×12400
2.—327×2400	7.—6275×30500
3.—562×1800	8.— 840×29000
4.—698×46000	9.—1504×20500
5.—849×57000	10.—3700×56800

65. Find the products of the following numbers:

1.—463×45	11.—90763×700
2.—348×62	12.—7422153×468
3.—793×86	13.—6929867×8000
4.—989×90	14.—9507340×7071
5.—$75 \times 42 \times 56$	15.—9264397×4762
6.—$84 \times 37 \times 69$	16.—1534693×9584
7.—7198×256	17.—9999999×9999
8.—93186×445	18.—$3854 \times 3854 \times 3854$
9.—99999×999	19.—1428576×70000
10.—76854×800	20.—7050860×70508

66. Multiply 657142 by each of the numbers from 56 to 81 inclusive.

67. Multiply 768354 by each of the numbers from 64 to 89 inclusive.

68. Multiply 876543 by each of the numbers from 78 to 103 inclusive.

69. Multiply 98998 by each of the numbers from 359 to 384 inclusive.

WRITTEN PROBLEMS.

70. There are 63 gallons in one hogshead. How many gallons are there in 25 hogsheads?

PROCESS.

63 gallons.
25
315
126
1575 gallons.

SOLUTION.—1. *Statement.*—In 25 hogsheads there are 25 times as many gallons as there are in 1 hogshead.

2. *Analysis.*—Since there are 63 gallons in one hogshead, in 25 hogsheads there are 25 times 63 gallons, which are 1575 gallons.

3. *Conclusion.*—Therefore, in 25 hogsheads there are 1575 gallons.

71. How many gallons are there in 36 hogsheads and 36 gallons?

72. How many gallons are there in 72 hogsheads and 54 gallons?

73. What is the cost of 48 hogsheads and 32 gallons of wine, at $3 a gallon?

74. There are 365 days in one year. How many days did Vice President Wilson live, who died at the age of 63 years?

75. A certain man arose at 5 o'clock each morning for 35 years; another arose at 7 during the same period. How many hours did the former gain upon the latter?

76. The average earnings of a certain gentleman during a period of 48 years was $6 per day. What were his receipts during that time?

77. There are 320 rods in one mile. What is the cost of 75 miles of road bed, at $75 a rod?

78. What is the cost of 176 miles of railroad, at $5485 a mile?

79. A gentleman purchased a farm of 468 acres, at $39 an acre, and soon after sold it at $47 an acre. How much did he gain?

80. Bought 175 head of horses at $127 each; 96 head of cattle at $56 each, and 328 hogs at $19 each; they were afterwards sold at a gain of $956. How much was received for them?

81. There are 2000 pounds in one ton of hay. How many pounds of hay can be cut from 12 acres of meadow that yields 3 tons an acre?

82. How much is the hay mentioned in the preceding example worth at $18 a ton?

83. Bought 356 acres of land, 237 acres of which is under cultivation, and worth $37 an acre, and the remainder is woodland, worth $29 an acre. I gave in part payment a house and lot worth $12150. How much is still owing?

84. Bought 12 house lots at $1680 each, and erected as many houses at a cost of $3875 each. What was the whole cost?

85. Purchased a farm of 360 acres, at $35 an acre, and sold 196 acres, at $36 an acre, and the remainder at $33 an acre. Did I gain or lose, and how much?

86. Bought a drove of 225 cattle at $25 a head; their cost of transportation was $1125. I afterwards sold them at $37 a head. How much did I gain?

87. Shipped 18 car loads of hogs, containing 28 hogs each, which I sold at $16 apiece. How much did I receive for them?

88. Bought a cargo of coal consisting of 378 tons, at $6.50 a ton, and sold 217 tons at $7.40, and the remainder at $8 a ton. How much do I make?

89. A gentleman's income is $9875.50 a year, and his

expenses a month are as follows: rent $125, household economy $135, and all other expenses $230. How much can he save each year?

90. If a gentleman's salary is $4000 a year, and he pays $36 a month for board, $198 a year for clothes, and $475 for all other expenses, how much can he save in 4 years?

91. There are 5280 feet in one mile. What is the cost of a railroad 168 miles and 3600 feet in length, at $3 a foot?

92. Purchased 9 lots at $1875 each, and erected upon them as many houses, at a cost of $4288 each. They were sold for $70000. What was my gain?

93. My bank stock consists of 4 $1000 bonds. All my other property, in cash, is worth 6 times as much as my bank stock. If I buy 358 acres of land at $36 an acre, how much will I have left?

94. A gentleman started upon a journey of 2000 miles, at the rate of 36 miles a day. After traveling 7 weeks and 4 days, omitting Sundays, what is the distance yet to be traveled?

95. In a certain army the number that came out of the battle was 12 times the number killed, which was 6436. The number wounded and missing was 3 times the number killed, less 9439. How many men went into the engagement?

96. A gentleman in making a journey around the world found that the distance traveled by vessel was 19 times as far as that traveled by rail, which was 1360 miles, and the distance by other conveyances was equal to that traveled by rail, less 1065 miles. What was the whole distance traveled?

97. What is the cost of 312 bushels of nuts, at 26c. a quart?

98. Find the cost of 78 hogsheads of wine, at 56c. a pint.

99. A clock ticks 3600 times in one hour. How often does it tick in 8 days and 3 hours?

100. In one cubic foot there are 1728 cubic inches. How many cubic inches in 96 cubic feet and 1245 cubic inches?

101. How far will a locomotive travel in 2 weeks, omitting Sundays, at the rate of 28 miles an hour for 8 hours each day?

102. What is the cost of 1675 chests of tea, each chest containing 67 pounds each, at $1.85 a pound?

103. Purchased 475 bushels of wheat at $1.38 a bushel, 96 bushels of corn at 56c. a bushel, and 157 bushels of oats at 65c. a bushel. What was the cost of all?

104. Bought a bill of goods as follows: 27 yards of cloth at $8.25 a yard, 56 yards of velvet at $9.80 a yard, 36 yards of silk at $2.75 a yard, and 125 yards of cotton cloth at 9c. a yard. How much was the cost of the goods?

105. How many minutes longer is the month of July than the month of June? How many minutes are there in June, July and August?

106. A house is worth $2450, the farm on which it stands is worth 12 times as much as the house, less $600, and the stock is worth twice as much as the house. What is the value of the house, farm and stock?

107. If a person receives an annual salary of $1875, and expends each year $312 for board, $105 for clothing, and $275 for other purposes, how much will he save in 18 years?

108. Purchased 58 tons of coal at $7.50 a ton, 96 cords of maple wood at $5.75 a cord, 128 cords of pine wood at $4.80 a cord. If I pay $1296 cash, how much remains to be paid?

109. Paid $136 for a wagon, 3 times as much for a yoke of oxen as for the wagon, and twice as much as for the oxen, less $265, for a span of horses. How much did they all cost?

110. A drover bought 356 beeves at $35 a head, 1235 sheep at $3.50 a head, and paid for their transportation to

market $975; he afterwards sold both the beeves and sheep for $21360. How much did he gain?

111. A merchant bought 63 pounds of sugar at 11c. a pound; 6 chests of tea, containing 78 pounds each, at $1.47 a pound; 2 hhd. of molasses, 63 gallons each, at 35c. a gallon; and 5 barrels of oil, 31 gallons each, at 28c. a gallon. What was the cost of the whole bill?

112. Bought 45 cases of boots, each containing 15 pairs, at $5.80 a pair; 36 cases of shoes, 24 pairs each, at $3.90 a pair; 24 cases of gaiters, 16 pairs each, at $3.25 a pair; and 56 pairs of slippers at $2.25 a pair. How much was the whole cost?

113. Bought a house, farm and the stock thereon; for the house $2197 was paid; the farm cost 5 times as much, less $1979, and the stock cost as much as the house and farm, less $4976. How much did they all cost?

114. A is worth $5466; B 7 times as much, less $1387; and C 3 times as much as A and B, less $2348. How much are they all worth?

115. If a man breathes 16 times a minute, how often will he breathe in one day, if there are 60 minutes in one hour, and 24 hours in one day?

116. What is the cost of 12 pounds 8 ounces of tea at 11c. an ounce, if there are 16 ounces in a pound?

117. What is the cost of a beam whose length is 12 yards 2 feet, at 75c. a foot?

118. There are 20 quires in one ream, and 24 sheets in one quire. What is the cost of 12 reams, 13 quires, and 17 sheets of note paper, at 2c. a sheet?

119. There are 100 pounds in one cwt. What is the cost of 18 cwt. 75 pounds of beef, at 6c. a pound?

120. Mr. B bought at one time 758 acres of land at $87.56 an acre, and at another time 96 acres at $67.48 an acre; he afterwards sold it all at $75.85 an acre. Did he gain or lose, and how much?

121. A capitalist who owned $2985 worth of bank stock,

and a farm worth 8 times as much, traded both for a factory worth $29750. How much remained to be paid?

122. A gentleman built 12 houses at a cost of $4725 each, and sold 8 of them at $5192 each, and the remainder at $4180 each. Did he gain or lose, and how much?

123. A steamboat travels 12 miles an hour. How far, at that rate, will the same boat travel in one week?

124. A merchant bought 45 pieces of cloth, each containing 28 yards, at $8 a yard. What was the cost?

125. A planter sold 125 bales of cotton, each weighing 345 pounds, at 27c. a pound. How much did he receive for the cotton?

126. A miller sold 650 barrels of flour, each weighing 196 pounds, at 5c. a pound; he took in part payment a yoke of oxen worth $250, and the rest in cash. How much cash did he receive?

127. A farmer owns 3 farms; each farm contains 165 acres; each acre produced 27 bushels of wheat, worth $2 a bushel. How much was the whole crop worth?

128. A bought of B 563 acres of land, at $48 an acre, and gave in payment a house worth $3756; a factory worth 6 times as much, less $1267, and the rest in money. How much money did A pay?

129. A horse is worth $96; the field in which he is pastured is worth 12 times as much; the whole farm is worth 9 times as much as the field; and the house $3216 less than the farm. How much is the house worth?

130. A is worth $2798; B 7 times as much, less $3289; C 12 times as much as B, plus $976; and D 8 times as much as A and B. How much are they all worth?

131. F., L. & Co. bought 85 pieces of silk, containing 48 yards each, for $10200, and sold it at $4 a yard. How many dollars was gained?

132. My house cost $1825; my farm cost $1200 more than 5 times the cost of my house. What is the cost of the house and farm?

SECTION VI.

DIVISION.

Art. 49. *1.* A gentleman divided 24 acres of land among his sons, giving to each son 6 acres. How many sons had he?

In this example it is proposed to divide 24 acres into a certain number of equal parts, each part of the size of 6 acres.

Since he divided 24 acres so as to give each son 6 acres, it is evident that there are as many sons as their are 6 acres in 24 acres. In 24 acres there are 4 times 6 acres.

Therefore, he had 4 sons.

50. *2.* A gentleman divided 24 acres of land equally among his 4 sons. How many acres did each son receive?

In this example it is proposed to divide 24 acres into 4 equal parts, so as to find the size of each part.

Since he divided 24 acres of land into 4 equal parts, each part is of that size of which it requires 4 of them to equal 24 acres; each of those parts is therefore one fourth of 24 acres, which is 6 acres; therefore, each son received 6 acres.

51. The first of these examples illustrates a form of the division of a quantity into a certain number of parts of a known size, and it will readily be perceived that this form of division is, in reality, only a short method of finding the remainder obtained by the continued successive subtractions of a series of equal numbers from some given number; as multiplication is a convenient method of finding the sum of several equal numbers.

For instance, in first example deduct one 6 acres, given to one son, and the remainder is 18 acres; in like manner deduct another 6 acres,

given to another son, from 18 acres, and the second remainder is 12 acres: again deduct a third 6 acres, given to a third son, from 12 acres, and the third remainder is 6 acres, which, given to the fourth son, leaves no remainder, and the number of times the known part, 6 acres, is deducted from the whole quantity, is the required number of parts.

It sometimes happens that the last remainder is greater than 0, but it is always less than the constant subtrahend.

Thus, 22−4=18; 18−4=14; 14−4=10; 10−4=6; 6−4=2; showing there are five 4's in 22, with a remainder, 2.

52. The second of these examples illustrates a form of the division of a quantity into a given number of equal parts, so as to ascertain the size of the parts.

Hence, Division is two-fold, as follows:

53. 1. ***Division*** is a process of finding the number of equal parts or numbers, into which a given number is separated, the size or part being known.

Thus, if 12c. be divided among a number of boys, giving each boy 3c., how many boys are there?

Here the whole quantity and the size of the parts are given to find the number of equal parts.

2. ***Division*** is a process of separating a given number into a required number of equal parts.

Thus, if 12c. be divided equally among 4 boys, how many cents will each boy receive?

Here the whole quantity and the number of equal parts are given to find the size of each part.

NOTE.—In all written operations the actual process in both forms of division is identical, though the forms of analysis of all examples in both oral and written problems are dependent upon the peculiar nature of the example.

54. The terms used in division are *dividend*, *divisor*, and *quotient*.

55. The ***Dividend*** is the number which contains a certain number of equal parts of a known size; or it is the number to be separated into a given number of equal parts.

Thus, in the first and second examples 24 acres is the dividend.

56. The ***Divisor*** is the number of equal parts or numbers into which the dividend is to be separated; or it is

one of the equal parts or numbers into which the dividend is to be separated.

Thus, in the first example 6 acres is the divisor, and in the second, 4 is the divisor.

57. The ***Quotient,*** in the *first form* of division, is the number of times the divisor is contained in the dividend; in the *second form* it is the value of one of the equal parts into which the dividend is to be separated.

Thus, in the first example, 4 is the quotient, and in the second, 6 acres is the quotient.

58. The ***Remainder,*** in division, is that part of the dividend which remains after separating it into a certain number of equal parts.

59. The ***Sign*** of ***Division*** is $\div$. When placed between numbers it signifies that the first number is to be divided by the second.

Thus, $36 \div 6 = 6$ is read 36 divided by 6 equals 6.

Division is also indicated by writing the dividend above, and the divisor below, a horizontal line; or by writing the divisor on the left, and the dividend on the right, of a vertical or curved line.

Thus, $\frac{36}{6} = 6$, $6 \mid 36 = 6$, $6)36 = 6$.

It will be noticed that in the second form of division the divisor denotes one of the equal parts into which the dividend is separated. The name of the parts being determined by the number of units in the divisor.

Thus, if the dividend be divided into *two* equal parts, each of the parts is called *one half* of the dividend; if into *three* equal parts, each part is called *one third* of the dividend; if into *eight* equal parts, each part is called *one eighth* of the dividend, etc.

The *one half* of a number is found by dividing the number by *two;* the *one third* of it is found by dividing it by *three*, and so on. In general, a given part of a number is found by dividing it by the number indicating the name of the parts.

60. In division, the divisor and quotient are factors of the dividend; as in multiplication the multiplicand and multiplier are factors of the product. It will be perceived,

therefore, that division is the converse of multiplication; in the latter the two factors are given to find the product; while in the former the dividend, corresponding to the product, and the divisor, corresponding to one of the factors, are given to find the quotient, corresponding to the other factor.

61. Division and multiplication are *proofs* of each other. *The product divided by a factor equals the other factor; or the product of the divisor and quotient equals the dividend.*

PRINCIPLES.

1. The dividend may be abstract or denominate.
2. If the dividend is denominate, either the divisor or quotient is denominate, but not both.
3. If the dividend is abstract, both divisor and quotient are abstract.
4. The remainder is always like the dividend.

ORAL PROBLEMS.

3. There are 4 quarts in one gallon. How many gallons are there in 32 quarts?

STATEMENT.—There are as many gallons in the given number of quarts as the number of quarts in one gallon is contained times in the given number of quarts.

ANALYSIS.—Since there are 4 quarts in one gallon, there are as many gallons in 32 quarts as 4 quarts are contained times in 32 quarts, which are 8 times.

CONCLUSION.—Therefore, in 32 quarts there are 8 gallons.

4. If $32 be equally divided among 4 persons, how much will each one receive?

STATEMENT.—Each person will receive such a part of the whole number of dollars as each person is a part of the given number of persons.

ANALYSIS.—Since $32 is divided equally among 4 persons, each person will receive ¼ of $32, which is $8.

CONCLUSION.—Therefore, each person will receive $8.

5. If $63 was paid for 9 yards of cloth, what is the cost of one yard?

6. A gentleman paid $72 for 12 head of sheep. What is the cost of one sheep?

7. There are 8 furlongs in one mile. How many miles are there in 72 furlongs?

8. Paid 96c. for 12 yards of ribbon. What is the cost of one yard?

9. If a man can dig 84 feet of trench in 7 days, how many feet can he dig in one day?

10. If 5 yards of silk cost $15, how much will 8 yards cost?

11. If 6 men can do a piece of work in 12 days, how long will it require 8 men to do the same?

12. How much will 7 yards of cloth cost if 8 yards cost $56?

13. What will be the cost of 20 oranges, if 9 oranges cost 45c.?

14. How many tons of coal at $9 a ton will pay for 12 barrels of flour at $6 a barrel?

15. A gentleman bought 15 yards of velvet at $4 a yard, and gave in payment $12, and the remainder in flour, at $6 a barrel. How many barrels did he give?

16. There are 12 ounces in one pound. How many pounds are there in 108 ounces?

17. If 8 quarts of molasses cost 72c., how much will $\frac{1}{3}$ of 15 quarts cost?

18. What must be paid to ride $\frac{1}{12}$ of 96 miles, if it cost 56c. to ride 7 miles?

19. If 9 men can mow 36 acres in one day, how much can 12 men mow?

20. If 6 men can build a boat in 14 days how long will it require 7 men to do it?

21. If 9 pounds of sugar cost 108c., how much will $\frac{1}{4}$ of a pound cost?

22. How much will 9 barrels of cider cost, if 5 barrels cost $45?

23. Gave $63 for 7 tons of coal. How much should be paid for 12 tons?

24. Bought 80 ounces of tea at $2 a pound. What is the cost?

25. Gave $\frac{1}{3}$ of $48 for 4 yards of silk. How much will $\frac{1}{4}$ of 24 yards cost, at the same rate?

26. If 12 boxes of oranges cost $72, how much will 8 boxes cost?

27. Paid 80c. for 4 quires of paper. How much should be paid for 9 quires?

28. If 6 men can do a piece of work in 15 days, in what time can 9 men do it?

29. How many men can do as much in 6 days as 5 men can in 12 days?

30. How many men can build a wall in 10 days, if 5 men can build it in 40 days?

31. What is the cost of $\frac{1}{2}$ of 16 yards of cloth, if 6 yards cost $48?

32. If 8 acres of land cost $320, how much will 12 acres cost?

33. What is the cost of $\frac{2}{3}$ of 15 pounds of coffee if 4 pounds cost 80c?

SUGGESTION.—$\frac{2}{3}$=twice as much as $\frac{1}{3}$. $\frac{1}{3}$ of 15 is 5, $\frac{2}{3}$ is two times 5, which are 10.

34. How much will $\frac{3}{4}$ of 12 yards of cashmere cost if 12 yards cost $36?

35. If 8 men can dig a trench in 20 days, how many men will be required to do it in 16 days?

36. What will be the cost of 9 yards of cloth if 7 yards cost $5.60?

37. What is the cost of $\frac{3}{5}$ of 16 bushels of wheat if 20 bushels cost $40?

38. If $\frac{1}{5}$ of 15 yards of ribbon cost 84c., how much will $\frac{1}{3}$ of 18 yards cost?

39. Bought 9 barrels of flour for $72, and gave 6 barrels for coal at $6 a ton. How many tons did I receive?

40. $\frac{3}{4}$ of \$40 is what I gave for 5 yards of cloth. What is the price of one yard?

41. $\frac{7}{8}$ of \$40 is \$5 less than what was paid for 8 sheep. What did one sheep cost?

42. Paid \$60 for 12 barrels of flour. At what price per barrel should it be sold so that \$12 may be gained?

43. I sold 12 dozen of pens for 84c, which was at a loss of 12c. How much did they cost per dozen?

44. What is the cost of $\frac{2}{3}$ of 18 sheep if $\frac{3}{4}$ of 12 sheep cost \$63?

45. A wagon cost \$60, which is 5 times the cost of the harness. What is the cost of both the harness and wagon?

46. A watch cost \$120, which is 4 times the cost of the chain. What is the cost of both watch and chain?

47. Paid \$96 for a sleigh, which was 8 times the cost of the bells, less \$5. What was the cost of the bells? Of both?

48. $\frac{4}{5}$ of \$60 is \$3 more than what was paid for 9 barrels of salt? What was the salt a barrel?

49. If 8 pounds of coffee cost \$1.60, how much will $\frac{3}{4}$ of a pound cost?

50. A tree is 72 feet high, which is 9 times the distance around it at the base. How many feet around the base?

51. Bought 12 yards of velvet at \$9 a yard, and sold it at a gain of \$24. What was the selling price a yard?

52. Paid \$75 for 25 sheep, and soon after sold them at a gain of \$25. How much was received for each?

53. Bought at one time 8 barrels of flour at \$7 a barrel, and 4 barrels at \$8 a barrel, and sold both lots so as to gain \$20. What was the selling price a barrel?

ORAL COMBINATIONS FOR DICTATION.

54. $12\times7+6\div9+12+5\div3\times4+8+5\div7\times5-15=?$

55. $9\times8\div6+9\div7+12\times3\div5+9-12\times8\div4\times5=?$

56. $56\div7+12-9\times7-7+11\div9\times6+2\div8\times7-19=?$

57. $63\div9\times6-12\div6+19+4\div7\times9-11\div5\times10=?$

58. $35+13\div6\times12+4\div10\times8+4\div7\times12-44\div10=?$

59. $9\times7-7\div7+7\times3+15\div5\times9+12\div12\times5\div2=?$

60. $15+9\div4\times8-12\div4\times8+3\div3\times4-50+6\div7=?$

61. $18+12\div5\times9+12\div6\times5-15\div5\times7-16-10=?$

62. $27\div3-9\times12+18\div2\times9-9\div9\times5+16\div7\times8=?$

63. $36+12-8\div8+10\times4-12\div8\times12+3\div3\times4=?$

64. $35-15\times4+4\div7\times4\div6\times8-4\div5\times3-16=?$

65. $27-17\times9-9\div9+9+12\div5\times7-12\div6+7=?$

66. $32-12+20\times2\div8\times6+12\div12\times7-12+6\div3=?$

67. $9+9+9\div3\times7-7\div7\times5+20\div5\times4-18=?$

68. $8+8+8+8\div4\times9\div6\times11-32\div10\times8+4\div7=?$

69. $81\div9\times7+3\div6\times11-21-4\div12\times7-16\div5\times4=?$

70. $7+12+6+8+7+9+11+12-8-7-6-9\div6=?$

71. $9+11+8+7\div5+8+7+8+9\div3+11\div3+8+8=?$

72. $12+18\div5+16+8+12\div6\times7+11\div5\times8-15=?$

73. $15+15+6\div6\times7+18\div6\times7+5+6\div9\times5\div9=?$

74. $144\div12+12+12+12\div6\times7+8\div8\times5+9\div7=?$

75. $121\div11\times7+7\div7\times4-6\div6\times8\div7\times6-12\div6=?$

76. $108\div9+7+7+7+7\div5\times9\div12\times10+40-50=?$

77. $50+50+100+200-300\div10\times7+7+7\div7\times6=?$

78. $20+20+40+16\div12\times7-16\div8+95+21\div11=?$

ANALYSIS OF DIVISION.

79. Divide 2563 by 6.

PROCESS.

$$\begin{array}{r|l} 6 & 2563 \\ & 24 \\ \hline & 16 \\ & 12 \\ \hline & 43 \\ & 42 \\ \hline & 1 \\ & 1 \\ \hline \end{array} \quad (427\tfrac{1}{6}$$

ANALYSIS.—1. Write the divisor at the left of the dividend, with a curved line between them, and for convenience begin to divide at the highest order of the dividend, thus:

2. 6 is contained in 2 thousands 0 thousands times, with a remainder of 2 thousands, which change to hundreds. 2 thousands equal 20 hundreds. 20 hundreds plus 5 hundreds equals 25 hundreds.

3. 6 is contained in 25 hundreds 4 hundred times, with a remainder of 1 hundred, which change to tens. 1 hundred equals 10 tens. 10 tens plus 6 tens equals 16 tens.

4. 6 is contained in 16 tens 2 tens times, with a

remainder of 4 tens, which change to units. 4 tens equals 40 units 40 units plus 3 units equals 43 units.

5. 6 is contained in 43 units 7 units times, with a remainder of 1 unit. Since 1 unit is less than 6, the division can only be indicated thus, $\frac{1}{6}$.

Hence, $2563 \div 6 = 427\frac{1}{6}$.

The above analysis illustrates that form of division in which it is required to find how often one number is contained in another of the same kind.

The following will illustrate the division of a number into a certain number of equal parts, and for this purpose it is assumed to find the $\frac{1}{6}$ of 2563, the written process being identical in both cases; thus,

Process.	Proof.
6) 2563 (427	
24	6
16	2562
12	1
43	2563
42	
1	

Analysis.—1. Write the numbers as before.

2. $\frac{1}{6}$ of 2 thousands is 0 thousands, with a remainder 2 thousands, which change to hundreds. 2 thousands equals 20 hundreds. 20 hundreds plus 5 hundreds equals 25 hundreds.

3. $\frac{1}{6}$ of 25 hundreds is 4 hundreds, with a remainder 1 hundred, which change to tens. 1 hundred equals 10 tens. 10 tens plus 6 tens equals 16 tens.

4. $\frac{1}{6}$ of 16 tens is 2 tens, with a remainder of 4 tens, which change to units. 4 tens is equal to 40 units. 40 units plus 3 units equals 43 units.

5. $\frac{1}{6}$ of 43 units is 7 units, with a remainder of 1 unit.

Hence, $\frac{1}{6}$ of 2563 is 427, with a remainder 1.

The preceding operations are by a process called "***Long Division.***" The following will serve to illustrate the same operation by a process called "***Short Division,***" which differs from the preceding in that the product of the divisor by each succeeding figure of the quotient, and the successive subtractions of these products are not written.

Thus, 6)2563

$427\frac{1}{6}$

Note.—Operations in short division are usually confined to those examples in which the divisor does not exceed 12.

WRITTEN EXERCISES.

80. Divide the following numbers by each of the numbers from 2 to 12 inclusive.

1.—54698	21.—590019	41.—1437691	61.—38670902
2.—21045	22.—467453	42.—3902914	62.—10456724
3.—42937	23.—370228	43.—7345678	63.—37936226
4.—82563	24.—995323	44.—6778898	64.—19977991
5.—63294	25.—201567	45.—7009304	65.—64951238
6.—58745	26.—554835	46.—9315952	66.—86817681
7.—54698	27.—863978	47.—7341169	67.—20290753
8.—21045	28.—354764	48.—4192093	68.—87654312
9.—73924	29.—822073	49.—4736587	69.—44332219
10.—36528	30.—323599	50.—9988776	70.—79911997
11.—49236	31.—765102	51.—4039217	71.—83215946
12.—54785	32.—358455	52.—5295139	72.—54321098
13.—78545	33.—745346	53.—7341196	73.—65431287
14.—36294	34.—915009	54.—9209314	74.—29075248
15.—92437	35.—674534	55.—5437678	75.—49512386
16.—69458	36.—455853	56.—8776988	76.—93626372
17.—65283	37.—702028	57.—9007304	77.—67093026
18.—45210	38.—746053	58.—5139592	78.—81768186
19.—64985	39.—651027	59.—4371167	79.—90752482
20.—82863	40.—807320	60.—5637478	80.—36263726

FIND THE QUOTIENTS OF THE FOLLOWING NUMBERS.

81. Divide 47836 by each of the numbers from 13 to 19, inclusive.

82. Divide 75048 by each of the numbers from 20 to 29, inclusive.

83 Divide 93840 by each of the numbers from 30 to 39, inclusive.

84. Divide 325000 by each of the numbers from 40 to 49, inclusive.

85. Divide 421648 by each of the numbers from 50 to 59, inclusive.

86. Divide 356405 by each of the numbers from 60 to 69, inclusive.

87. Divide 483706 by each of the numbers from 70 to 79, inclusive.

88. Divide 563848 by each of the numbers from 80 to 89, inclusive.

89. Divide 7056845 by each of the numbers from 90 to 99, inclusive.

90. Divide 8325436 by each of the numbers from 100 to 109, inclusive.

91. Find $\frac{1}{4}$ of 346228; 453764; 718296; 2937573.

92. Find $\frac{1}{5}$ of 423675; 576840; 637385; 8536410; 701275; 874600; 965435; 1287595.

93. Find $\frac{1}{6}$ of 732848; 687256; 843628; 9427638; 354982; 464844; 578976; 8749674.

94. Find $\frac{1}{7}$ of 385363; 785449; 679384; 8765477; 499375; 589754; 691866; 7082943.

95. Find $\frac{1}{9}$ of 784684; 783627; 874654; 9763481; 587816; 679436; 738848; 9353976.

96. Divide 543284 by 128; by 156; by 189.

97. Divide 6378424 by 236; by 352; by 475.

98. Divide 4863745 by 375; by 468; by 584.

99. Divide 7548638 by 488; by 557; by 639.

100. Divide 8376547 by 654; by 729; by 856.

101. Divide 6745438 by each of the numbers from 371 to 390, inclusive.

102. Divide 7438456 by each of the numbers from 452 to 470, inclusive.

103. Divide 8436782 by each of the numbers from 654 to 667, inclusive.

104. Divide 5337846 by each of the numbers from 829 to 842, inclusive.

105. Divide 6007408 by each of the numbers from 947 to 965, inclusive.

WRITTEN PROBLEMS.

106. There are 24 grains in one pennyweight of gold. How many pennyweights are there in 864 grains?

PROCESS.

24) 864 (36
72
144
144

SOLUTION.—1. *Statement.* There are as many pennyweights in the whole number of grains as the number of grains in one pennyweight is contained times in the whole number of grains.

2. *Analysis.* Since there are 24 grains in one pennyweight, in 864 grains there are as many pennyweights as 24 grains is contained times in 864 grains, which are 36 times.

3. *Conclusion.* Therefore, in 864 grains there are 36 pennyweights.

107. Paid $1225 for 25 head of cattle. What was the cost of one head?

PROCESS.

25)$1225($49
100
225
225

SOLUTION.—1. *Statement.* The cost of one head is such a part of the whole cost as one head is a part of the whole number of cattle.

2. *Analysis.* Since 25 head of cattle cost $1225, one head costs $\frac{1}{25}$ of $1225, which are $49.

3. *Conclusion.* Therefore, one head of cattle costs $49.

108. There are 20 pennyweights in one ounce of silver. How many ounces are there in 1260 pennyweights?

109. There are 32 quarts in one bushel. How many bushels are there in 2496 quarts?

110. Gave $3321 for 27 acres of land. What is the price per acre?

111. Traveled 1088 miles in 28 days. What was the distance traveled per day?

112. A gentleman paid $4500 for 36 horses. What is the price of each horse?

113. There are 63 gallons in one hogshead. How many hogsheads are there in 12474 gallons?

114. How many years are there in 11700 weeks, if there are 52 weeks in one year?

115. How many cubic yards are there in 6912 cubic feet, if there are 27 cubic feet in one cubic yard?

116. If there are 320 rods in one mile, how many miles are there in both sides of a roadway 6080 rods in length?

117. If each rail of a railroad track is 30 feet in length, how many rails will be required to lay a track 5280 feet?

118. 3 times \$16000 is what was paid for 12 houses. What is the cost of each house? What is the cost of 7 houses?

119. A, B, and C join together to purchase flour. A contributed \$1800; B twice as much as A, and C as much as A and B, less \$900. How many barrels of flour can they purchase at \$9 a barrel?

120. If 17 cows are worth \$816, what is one cow worth? How much are 48 cows worth?

121. Gave \$4224 for 32 acres of land. At the same rate, what is the cost of one acre? Of 24 acres?

122. If 75 horses are worth \$11100, how much, at the same rate, are 56 horses worth? 63 horses?

123. If 18 acres of land are worth \$3366, for how much per acre should it be sold so that \$720 may be gained?

124. How many tons of coal at \$9 a ton will pay for 84 thousand feet of lumber at \$36 per thousand feet?

125. By selling 31 acres of land for \$3100, I lose \$155. What is the cost per acre? Cost of 16 acres?

126. If 180216 square rods of land be divided equally among 12 men, what is the value of each man's share at \$1.25 per square rod?

127. $\frac{1}{9}$ of \$42075 is what a gentleman paid for a house and lot, which was \$568 more than what he sold them for. How much was received for them?

128. \$36864 is 12 times what I paid for 96 acres of land. What is the cost per acre? Of 35 acres?

129. How many days will 128200 pounds of flour last 641 men, giving each man 4 pounds each day?

130. A merchant sold 63 barrels of oil for \$1008, gaining \$126. How much did the oil cost per barrel? Cost of 36 barrels?

131. By selling 145 tons of hay for \$2320, I lost \$435. What did one ton cost? Cost of 48 tons?

132. Bought 96 horses for \$12000, and sold 78 of them at \$136 each, and the rest at cost. How much did I gain?

133. Sold 48 acres of land for \$5520 and gained \$480. What was the cost of 27 acres?

134. Bought 42 carriages for \$5376. For how much should I sell 18 of them so as to gain \$126?

135. If I sell 124 head of cattle for \$3720, and lose \$620, for how much should I sell 56 head to gain \$372?

136. A, B, C, and D, wish to buy coal; A and B each furnish \$2500; C furnishes as much as both A and B; and D furnishes as much as the other three. How much coal at \$8 a ton can they buy?

137. If \$7560 be divided so that A shall receive \$175 more than $\frac{1}{3}$ of it, B \$1250 less than $\frac{1}{2}$ of it, and C the remainder, how much will each one receive?

138. A farmer sold a grocer 20 pounds of butter, at 18c. a pound; 17 dozen eggs, at 12c. a dozen; 9 bushels of potatoes, at 60c. a bushel, and received in payment 54 pounds of sugar, at 14c. a pound, and the remainder in rice, at 12c. a pound. How many pounds of rice did he get?

139. A gentleman owned $\frac{1}{12}$ of a tract of land of 12960 acres, and divided it equally among his 8 children. What is the value of each child's share, at \$36 an acre?

140. If the divisor is 341, and the quotient 589, what is the dividend?

141. From the sum of 49 thousand 5, and 3 thousand 2, subtract their difference, and divide the remainder by 204.

142. A grain dealer delivered to a customer 3240 pounds of wheat. If there are 60 pounds in one bushel, how many bushels did he deliver?

143. If 48 bushels of oats weigh 2304 pounds, how many pounds will 75 bushels weigh? How many pounds will a car-load of 536 bushels weigh?

144. A, B, and C, bought a farm for \$9580; A contributed \$125 less than $\frac{1}{4}$ of the whole sum; B \$1584 more than $\frac{1}{5}$ of it, and C the remainder. How much did each one contribute?

145. Bought a tract of land and divided it into 72 house

lots, which I sold at $248 each, thereby gaining $6912. What was the cost per lot?

146. B, C, and D, purchased 264 acres of land for $11880; B's share of it was 12 acres more than $\frac{1}{3}$ of it; C's share 24 acres more than $\frac{1}{4}$ of it, and D's the remainder. How much did each one pay?

147. If two poems contain respectively 15693 and 9892 lines, in how many days can a boy read both of them if he reads 85 lines each day?

148. A sold to a merchant 2460 pounds of wool at 45c. a pound, 1840 pounds at 36c. a pound; received $219.40 cash, and the remainder in silk at $5 a yard. How many yards of silk did he receive?

149. A fruit dealer sold 85 barrels of nuts, each containing 3 bushels, at $8 a bushel, and gained $255. What was the cost a barrel?

150. A farmer picked from one tree 320 apples; if he has 5 trees bearing the same number of apples, how many barrels can be filled from them, allowing 3 bushels to one barrel, and 40 apples to one peck? How many bushels will remain?

151. A man sold 64 calves at $7 each, and 29 sheep at $13 each, and with the money received bought 33 barrels of syrup. How much did he pay for four barrels?

152. 87 cattle were found to weigh 34539 pounds. If they were of equal weight, how much more are 47 of them worth at 15c. a pound, than the remainder at 17c. a pound?

153. A miller earns $4.50 a day, and his expenses are $40 a month. How many months, omitting Sundays, will it take him to pay for a horse and wagon worth $136, and a mortgage on his house of $476?

154. A chaise wheel makes 349 revolutions in one mile. What is the distance traveled when the wheel has made 17101 revolutions? What is the time consumed at 7 miles an hour?

155. A colony of 121 persons contribute $945 each towards the purchase of land. How many acres, at $21 an acre, can they buy?

156. A brickmaker has 3 kilns, one of which contains 36520 bricks, and each of the others $\frac{1}{4}$ as many; after using 7910 bricks, he sold the remainder in 43 loads. How many bricks are there in each load?

157. Find the product of the sum and difference of four thousand ninety, and eight hundred seventy, and divide it by 905.

158. Two drovers each have 954 sheep, costing $7 a a head; one of them sold them at a loss of $1 a head, and the other at a gain of $2 a head. How many sheep of an extra quality, valued at $30 a head, can they jointly buy with the proceeds of the sale?

159. If the quotient is 17 when the divisor is 27, what will be the dividend if the quotient remain the same, and the divisor be doubled?

160. There are 5280 feet in one mile. How often will a wheel 9 feet in circumference turn around in a distance of 18 miles?

161. If a certain number be multiplied by 16, the product is 400; what is the product of the same number if multiplied by 192?

162. If a certain number be divided by 125, the quotient is 48; what is the quotient if the same number is divided by 25?

163. The divisor is 136, the quotient 98, and the remainder 105. What is the dividend?

164. What number divided by 528 will give 36 for the quotient, and 44 for the remainder?

165. What number multiplied by 86 will give the same product as 430 multiplied by 163?

166. If 17 acres of land be sold at $28 an acre, and thereby $119 is lost, what is the cost of 29 acres at the same rate?

TO DIVIDE BY 10, 100, OR 1000.

Any integral number may be divided by 10, by 100, by 1000, etc., by pointing from the right of the dividend as many figures as there are ciphers in the divisor.

Thus, $2846 \div 10 = 284.6 = 284$ and 6 *tenths*.
$2846 \div 100 = 28.46 = 28$ and 46 *hundredths*.
$2846 \div 1000 = 2.846 = 2$ and 846 *thousandths*.

1.—Divide 349 by 10.
2.—Divide 867 by 100.
3.—Divide 3468 by 100.
4.—Divide 7962 by 1000.
5.—Divide 6459 by 100.
6.—Divide 8678 by 100.
7.—Divide 18469 by 100; by 1000.
8.—Divide 6984 by 100.
9.—Divide 1649 by 100; by 1000.
10.—Divide 2564 by 1000.

Removing the decimal point one order to the right *multiplies* the number by 10; removing the point two orders to the right multiplies by 100.

Thus, $34.56 \times 10 = 345.6$; $34.56 \times 100 = 3456$.

Removing the decimal point one order to the left *divides* the number by 10; removing the point two orders to the left divides by 100.

Thus, $678.9 \div 10 = 67.89$; $678.9 \div 100 = 6.789$.

If a divisor ends in one or more ciphers, point from the right of the dividend as many figures as there are ciphers on the right of the divisor; divide what is left of the dividend by the significant figures of the divisor. To the remainder, if any, annex the figures separated from the right of the dividend.

1.—Divide 643 by 90.

PROCESS.

$643 \div 90 = 64.3 \div 9 = 7\frac{1}{9}\frac{3}{0}$.

ANALYSIS.—Point the 3 from the right of the dividend, and divide 64 by 9. $64 \div 9 = 7$, and 1 remainder. To this 1 annex the separated 3, making 13. The quotient is 7, with 13 remainder; or, the complete quotient is $7\frac{1}{9}\frac{3}{0}$.

2.—Divide 345 by 60.
3.—Divide 8469 by 70.
4.—Divide 3792 by 500.
5.—Divide 7264 by 600.
6.—Divide 84692 by 2400.
7.—Divide 96728 by 3500.
8.—Divide 66729 by 4800.
9.—Divide 487629 by 23000.

SECTION VII.

UNITED STATES MONEY.

Art. 62. ***Currency*** is that which is given and taken as having value and representing property, and consists of *coin* or *specie*, and *paper money*.

63. ***Coin*** or ***Specie*** is metal stamped, and authorized by law to be used as money.

64. ***Paper Money*** consists of *notes* issued by the Treasury of the United States, or by banks, as substitute for coin.

65. A ***Decimal Currency*** is a currency whose denominations increase and decrease in a tenfold ratio.

66. ***United States Money*** is a decimal currency, and is often called *Federal Money;* it was adopted by Congress in the year 1786, as the currency of the United States.

67. The coin of the United States consists of gold, silver, nickel, and bronze.

68. The ***Gold*** coins are the one-dollar, three-dollar, quarter-eagle, half-eagle, and double-eagle pieces.

69. The ***Silver*** coins are the trade-dollar, half-dollar, quarter-dollar, and ten-cent pieces.

70. The ***Nickel*** coins are the three-cent and the five-cent pieces.

71. The ***Bronze*** coins are the one-cent pieces.

NOTE. — The silver five-cent and three-cent pieces, and the bronze two-cent pieces, are no longer coined.

72. The *gold coins* are made of 9 parts pure gold and

1 part alloy, consisting of silver and copper. The *silver coins* are made of 9 parts pure silver and 1 part copper. The *nickel coins* are made of 75 parts copper and 25 parts nickel. The *bronze coins* are made of 95 parts copper and 5 parts zinc and tin.

TABLE OF UNITED STATES MONEY.

10 mills (m)	-	equal 1 cent, ¢
10 cents,	- -	" 1 dime, d.
10 dimes, or 100 cents,		" 1 dollar, $
10 dollars,	- -	" 1 eagle, E.

TABLE OF PARTS OF A DOLLAR.

$\frac{1}{2}$ dollar = 50 cents.	$\frac{2}{3}$ dollar = $66\frac{2}{3}$ cents.
$\frac{1}{3}$ dollar = $33\frac{1}{3}$ cents.	$\frac{3}{4}$ dollar = 75 cents.
$\frac{1}{4}$ dollar = 25 cents.	$\frac{2}{5}$ dollar = 40 cents.
$\frac{1}{5}$ dollar = 20 cents.	$\frac{3}{5}$ dollar = 60 cents.
$\frac{1}{6}$ dollar = $16\frac{2}{3}$ cents.	$\frac{4}{5}$ dollar = 80 cents.
$\frac{1}{8}$ dollar = $12\frac{1}{2}$ cents.	$\frac{3}{8}$ dollar = $37\frac{1}{2}$ cents.
$\frac{1}{10}$ dollar = 10 cents.	$\frac{5}{8}$ dollar = $62\frac{1}{2}$ cents.
$\frac{1}{12}$ dollar = $8\frac{1}{3}$ cents.	$\frac{7}{8}$ dollar = $87\frac{1}{2}$ cents.

ORAL.

1. How many cents in one half a dollar? In a quarter of a dollar?

2. How many cents in one eighth of a dollar? In $\frac{3}{8}$ of a dollar?

3. How many cents in $\frac{3}{4}$ of a dollar? In $\frac{5}{8}$ of a dollar?

4. How many cents in $\frac{1}{3}$ of a dollar? In $\frac{2}{3}$ of a dollar?

5. What part of a dollar is 50 cents? 25 cents?

6. What part of a dollar is 75 cents? $12\frac{1}{2}$ cents? $37\frac{1}{2}$ cents?

7. What part of a dollar is $62\frac{1}{2}$ cents? $87\frac{1}{2}$ cents? $33\frac{1}{3}$ cents? $66\frac{2}{3}$ cents?

8. How many cents in one tenth of a dollar? 3 tenths of a dollar?

9. How many cents in $\frac{1}{5}$ of a dollar? $\frac{3}{5}$ of a dollar?

NOTE.—The teacher should give thorough drill upon the aliquot parts of a dollar.

73. The sign $ is written on the left of the figures which express dollars.

Thus, $25 is read *twenty-five dollars.*

74. Cents are hundredths of a dollar, and, when written *decimally*, occupy the two decimal places on the right of the *decimal point*, which separates cents from dollars.

Thus, $9.25 is read *nine dollars twenty-five cents.*
$6.08 is read *six dollars eight cents.*
$ .50 is read *fifty cents.*

In these cases cents are regarded as hundredths of the dollar, which is the unit. When the *cent* is regarded as the *unit*, no decimal point is necessary: as 10 cents, 125 cents, 5c., 8c., etc.

75. Mills occupy the third decimal place on the right of the decimal point.

Thus, $1.125 is read *one dollar twelve cents five mills;*
Or, *one dollar twelve and one half cents.*
$ .005 is read *five mills*, or *one half cent.*

WRITTEN.

10. Add four dollars fifty cents; eleven dollars twenty five cents; 30 dollars 40 cents; 7 dollars 13 cents.

11. Add 8 mills; 20 cents; 1 dollar; 2 dollars 30 cents; twelve cents five mills.

12. $5+$3.75+$84.00+$.15+$.125+$1.05+$.008=?

13. $1.75+$2.50+$3.25+$1.125+$.75+$.375?=

14. $10\frac{1}{2}+$2\frac{4}{5}+$7\frac{3}{4}+$2\frac{1}{2}+$4.00=?$

15. $8\frac{1}{4}+$3.75+$9\frac{3}{5}+$3.25+$8.625+$.87\frac{1}{2}=?$

16. $400+$961+$185.00+$37.50+$483.40+$175=?

17. $35+$35.00+$3500+$.35+$3.50+$.035=?

18. \$184 − \$35 = ?
19. \$32.50 − \$28.75 = ?
20. \$1340 − \$875.50 = ?
21. \$9837.40 − \$1568.90 = ?
22. \$235.92 − \$146.875 = ?
23. \$1000 − \$48.375 = ?
24. \$846.59 − \$150 = ?
25. \$137.945 − \$84.30 = ?
26. \$12.75 − \$1.875 = ?
27. \$93.625 − \$85½ = ?
28. \$18465½ − \$387.25 = ?
29. \$9643.80 − \$4673⅘ = ?

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30. \$1375 × 18 = ?
31. \$1200 × 29 = ?
32. \$83.75 × 34 = ?
33. \$96.875 × 56 = ?
34. \$864.50 × 60 = ?
35. \$12.375 × 75 = ?
36. \$39.40 × 12½ = ?
37. \$324.60 × 37½ = ?
38. \$596.25 × 33⅓ = ?
39. \$95.625 × 62½ = ?
40. \$194.00 × 100 = ?
41. \$137.255 × 125 = ?

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42. \$1000 ÷ 4 = ?
43. \$3750 ÷ 25 = ?
44. \$9640 ÷ 80 = ?
45. \$58.50 ÷ 48 = ?
46. \$1.375 ÷ 24 = ?
47. \$5.625 ÷ 48 = ?
48. \$3275.60 ÷ 48 = ?
49. \$5973.64 ÷ 124 = ?
50. \$375.00 ÷ 45 = ?
51. \$250.50 ÷ 50 = ?
52. \$182.04 ÷ 64 = ?
53. \$39.406 ÷ 124 = ?

BILLS.

76. A ***Bill*** of ***Goods*** is a written statement of articles sold, with the date of sale, the price or value annexed to each article, and is given by the seller to the buyer.

77. An ***Account*** is a written statement of *debits* and *credits* in business, as between two parties, the buyer and the seller.

78. A ***Debit*** is a written record of what is due or expected from the buyer to the seller.

79. A ***Debtor*** is the party from whom payment is due or expected.

80. A ***Credit*** is a written record of payment, in part or in full, made by the debtor.

81. A ***Creditor*** is the party to whom payment is due, or by whom it is expected.

82. When payment is made in full of a bill, it is *receipted*, and is then called a *receipted bill*.

NOTE.—The following are simple forms of bills, serving as models, to which the teachers should add daily from dictation until pupils are familiar with the business forms.

1. Chicago, Jan. 1, 1876.

James Watson,

To GEO. SHERWOOD & Co., *Dr.*

1875.							
Oct. 1	To 4 doz. Model First Readers, -	@ $3.50	$14	00			
Nov. 10	" 5 doz. Model Second Readers,	@ $4.00	20	00			
Nov. 30	" 12 doz. Writing Spellers, - -	@ $1.10	13	20			
Dec. 20	" 12 doz. No. 2 High School Slates,	@ $1.50	18	00	$65	20	

Received payment,

GEO. SHERWOOD & Co.
Per Sprague.

2. Cincinnati, Jan. 25, 1876.

Theodore Smith,

Bo't of WILSON, HUKILL & Co.

32 pieces of Brussels Carpet, 56 yards each, @ $3.80 . . .
48 pieces of Wilton " 64 yards " @ 3.25 . . .
50 pieces of Turkey " 63 yards " @ 3.40 . . .
64 pieces of Ingrain " 48 yards " @ 1.75 . . .

3. Columbus, O., April 1, 1876.

Mr. George Caldwell,

To HILL, JOHNSON & Co., *Dr.*

1876.
Jan. 1. To 36 yards of Linen, @ $2.50 . . .
Feb. 1. " 40 yards of Cambric, @ 60c. . .
March 1. " 60 yards of Cloth, @ 8.00 . . .
March 15. " 24 yards of Velvet, @ 16.00 . . .

4. St. Louis, June 1, 1876.

Mrs. James Shields,

Bought of MILLS, BROWN & Co.

1876.			
April 15.	24 yards of Cloth, -	@	$3.80. . .
April 25.	13 yards of Velvet,	@	9.00. . .
May 10.	36 yards of Silk, -	@	4.50. . .
May 25.	27 yards of Cashmere,	@	2.75. . .

5. Springfield, Ill., July 1, 1876.

Mr. Thomas Barnes,

Bought of STONE, ANDERSON & Co.

1876.			
May 28.	24 tons of Lackawanna Coal,	@	$7.50. . .
May 30.	30 tons of Briar Hill Coal,	@	7.00. . .
June 12.	45 cords of Maple Wood,	@	8.00. . .
June 18.	36 cords of Beech Wood,	@	7.50. . .

6. Madison, Wis., June 25, 1876.

Messrs. Paul, Everett & Co.,

To WILLIAM NELSON, *Dr.*

1876.			
Jan. 10.	To 125 barrels of Flour,	@	$7.80 . . .
Jan. 18.	" 236 bushels of Wheat,	@	1.20 . . .
Feb. 5.	" 190 bushels of Corn,	@	90c. . .
" "	" 80 sacks of Potatoes,	@	1.80 . . .
March 20.	" 48 sacks of Corn Meal,	@	2.75 . . .

REVIEW QUESTIONS.

What is the *sum* of several numbers? What is *addition?* What sign is used to indicate addition? What are *addends?* Describe the sign of *equality?* What is the *principle* applicable to addition of numbers? How should numbers be written for addition? At which order begin the operation? How may errors in addition be easily detected?

What is meant by the *difference* of two numbers? Define *subtraction*. What terms are employed in subtraction? Define *minuend*. Define *subtrahend*. What sign is used to indicate subtraction? When is the difference called remainder? What is the principle applicable to subtraction of numbers? How should numbers be written for subtraction? At which order begin the operation? If a figure in any order of the minuend is less than the figure in the same order of the subtrahend, how is the deficiency supplied? What is a proof of subtraction?

What is *multiplication?* By what other process may the same result be produced? What terms are used in multiplication? Define *multiplicand*. Define *multiplier*. Define *product*. What sign is used to indicate multiplication? What are the multiplicand and multiplier often called? What is the principle first named applicable to multiplication? Second principle? Third principle? Fourth principle? Of what denomination is the product? What is meant by *partial product?* What are the proofs of multiplication? How is the continued product of several numbers indicated? Mention two other methods of indicating the product of several numbers.

How many forms of *division* are there? Define first form. Define second form. What difference of written operation in the two forms? What determines the particular form in oral and written analyses of examples? What are the terms used in division? Define *dividend*. Define *divisor*. Define *quotient*. Define *remainder*. How does the remainder compare with the divisor? What sign is ordinarily used to indicate division? In what other ways may division be indicated? How is division related to multiplication? With reference to the dividend, what are the divisor and quotient called? What is the proof of division? What is the first principle applicable to division? Second principle? Third principle? Fourth principle? In second form of division, how is the given part of the dividend found? At which order begin to divide? Of what denomination is the remainder? What is meant by *short division?*

What is *currency?* Of what does it consist? Describe *coin* or *specie*. Define *paper money*. What is a *decimal currency?* Define United States money. By whom and when was it established as the U. S. currency? Of what does the coin of the U. S. consist? Name the gold coins. Name the silver coins. Name the nickel coins. Name the bronze coins. What is the composition of the gold coins? Silver coins? Nickel coins? Repeat the table of U. S. money.

What is a *bill* of *goods?* An *account?* Define *debit*. A *debtor*. Define *credit*. A *creditor*. What is a receipted bill?

SECTION VIII.

PROPERTIES OF NUMBERS.

FACTORS.

The product of 3 times 4 is 12; 3 and 4 each is therefore called a *factor* of 12. Hence,

Art. 83. A ***Factor*** of a number is one of the integers whose product is that number.

It is also called a *divisor* or *measure* of the number.

84. The factors of a number may be unequal as the above, or they may be equal, if unity be excluded, whose use does not affect the product.

As, 2=2; the product of 2 times 2 is 4; the product of 2 times 2 times 2 is 8. It will be seen that 2 is the *only* factor of 2, is one of the *two* equal factors of 4, and one of the *three* equal factors of 8. It is therefore called a *root* of 2, 4 or 8. Hence,

85. A ***Root*** of a number is the number itself, or one of the *equal* factors of the number.

The root is designated as *first*, *second*, *third*, etc., root, according to the number of times it is used as a factor.

86. The ***First Root*** of a number is the number itself.

87. The ***Second Root*** of a number is *each* of the *two equal* factors of a number.

The second root is usually called the ***Square Root.***

Thus, 3 is the square root of 9, since 9 is the product of 3×3.

88. The ***Third Root*** of a number is *each* of the *three* equal factors of a number.

The third root is usually called the ***Cube Root.***

Thus, 2 is the cube root of 8, since 8 is the product of $2\times2\times2$.

89. There are numbers that have no factors but themselves and unity.

As, 3, whose factors are 3 and 1; 5, whose factors are 5 and 1; etc. Such numbers are called *prime* numbers. All other numbers are the product of factors each greater than unity; as, 15 is the product of 3 and 5, 24 is the product of 4 and 6. Such numbers are called *composite* numbers. Hence,

90. Numbers, considered with respect of their factors, are *prime* or *composite*.

91. A ***Prime Number*** is a number whose only factors are itself and unity.

92. A ***Composite Number*** is the product of two or more integers, each greater than unity.

93. A ***Prime Factor*** of a number is a factor which is a prime number.

Thus, 2 and 3 are each prime factors of 6 or 12.

94. A ***Composite Factor*** of a number is a factor which is a composite number.

Thus, 4 and 6 are each composite factors of 12 or 24.

Since 2 and 3 are each factors of 6 and 12, they are called *common factors* of those numbers. Hence,

95. A ***Common Factor*** of two or more numbers is a factor of each of the numbers.

96. Numbers that have no *common* factor greater than unity are *prime* to each other.

Thus, 4 and 5; 8 and 9 are prime to each other.

97. One number is *divisible* by another, when the latter is a factor of the former.

Thus, 3 being a factor of 6, 6 is divisible by 3.

98. ***Even*** numbers are numbers which are divisible by 2.

Thus, 2, 4, 6 and 8, are even numbers.

99. ***Odd*** numbers are numbers which are not divisible by 2.

Thus, 3, 5, 7 and 9, are odd numbers.

100. Factoring is the method of obtaining the several factors or divisors of a number.

PRINCIPLES.

1. Every number is equal to the product of all its prime factors.

2. A factor of a number is also a factor of any product of that number by integers.

3. A common factor of several numbers is a factor of their sum.

4. A common factor of two numbers is a factor of their difference.

101. To resolve a number into its prime factors.

What are the prime factors of 60?

Process.

$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ \hline & 5 \end{array}$$

Analysis.—By trial, 60 divided by the prime factor 2 gives the composite factor 30 for a quotient; 30 divided by the prime factor 2 gives the composite factor 15 for a quotient; 15 divided by the prime factor 3 gives the prime factor 5 for a quotient. Hence, 2, 2, 3, and 5, are the prime factors of 60.

Rule.—*Divide the number by its least prime factor, and divide this and each succeeding quotient in the same manner until a quotient is obtained which is a prime number. The several divisors and the last quotient are the prime factors of the numbers.*

What are the prime factors of the following numbers?

1.	18.	*11.*	72	*21.*	480.
2.	24.	*12.*	81.	*22.*	550.
3.	27.	*13.*	84.	*23.*	585.
4.	32.	*14.*	96.	*24.*	672.
5.	42.	*15.*	108.	*25.*	512.
6.	45.	*16.*	121.	*26.*	729.
7.	49.	*17.*	144.	*27.*	756.
8.	54.	*18.*	240.	*28.*	860.
9	56.	*19.*	360.	*29.*	975.
10.	63.	*20.*	475.	*30.*	1200.

What prime factors are common to the following numbers?

31. 15 and 18.	*36.* 42 and 48.
32. 18 and 24.	*37.* 42 and 49.
33. 27 and 36.	*38.* 56 and 72.
34. 35 and 45.	*39.* 63 and 81.
35. 36 and 45.	*40.* 72 and 84.

CANCELLATION.

102. Cancellation is the process of facilitating operations in division by rejecting common factors from both dividend and divisor.

The ***Sign*** of ***Cancellation*** is an oblique mark drawn across the number in which is the canceled factor.

Thus, $\not{3}$, $\not{5}$, $\not{6}$, $\not{9}$.

103. The operations in cancellation are dependent upon the following

PRINCIPLES.

1. The canceling of one of the factors of a number divides that number by the factor canceled.

2. The canceling of one of a series of factors divides their product by the factor canceled.

3. The canceling of equal factors from both dividend and divisor does not change the value of the quotient.

1. Divide $3\times6\times8\times12$ by $2\times3\times4\times6$

PROCESS.

$$\frac{\overset{4}{\not{3}}\times\not{6}\times\not{8}\times12}{\underset{}{\not{2}\times\not{3}\times\not{4}\times\not{6}}}=12.$$

ANALYSIS.— The factors of the dividend being arranged above, and the factors of the divisor beneath a horizontal line, 3 is observed to be factor in both dividend and divisor, and is canceled from both, since by principle 3 the value of the quotient is not changed.

In like manner, 6 is observed to be a factor of both dividend and divisor, and is canceled. 2, a factor common to 2 of the divisor and 8 of the dividend, is canceled, leaving 4 the other factor of 8, in the dividend.

4, a factor of both dividend and divisor, is canceled, leaving 12 of the dividend the only factor uncanceled, which is therefore the quotient. Hence the following

Rule.—I. *Write the factors of the dividend above and the factors of the divisor beneath a horizontal line.*

II. *Cancel equal factors of both dividend and divisor until the factors of the one are prime to the factors of the other.*

III. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

2. Divide $4\times6\times8\times10$ by $2\times3\times4\times5$.

3. Divide $5\times7\times9\times11$ by $3\times5\times7\times11$.

4. Divide $6\times8\times12\times18$ by $4\times6\times9\times12$.

5. Divide $9\times12\times15\times18$ by $6\times5\times9\times12$.

6. Divide $10\times14\times21\times28$ by $3\times7\times4\times14$.

7. Divide $15\times20\times25\times27$ by $10\times15\times18\times25$.

8. Divide $18\times24\times32\times36$ by $9\times12\times16\times18$.

9. Divide $20\times30\times35\times40$ by $10\times15\times7\times8\times5$.

10. Divide $24\times28\times36\times45$ by $16\times18\times15\times12$.

11. Divide $21\times24\times28\times35$ by $7\times14\times24\times5$.

12. Divide $24\times30\times36\times40$ by $20\times15\times18\times12$.

13. $(27\times32\times36\times45)\div(16\times9\times12\times15)=?$

14. $(28\times30\times35\times40)\div(14\times15\times20\times7)=?$

15. $(30\times35\times40\times45)\div(15\times20\times30\times5)=?$

16. $(32\times36\times48\times54)\div(16\times18\times32\times27)=?$

17. $(36\times42\times45\times50)\div(18\times21\times15\times25)=?$

18. $(35\times45\times50\times55)\div(15\times25\times35\times11)=?$

19. $(40\times48\times56\times60)\div(20\times24\times28\times40)=?$

20. $(45\times63\times72\times80)\div(15\times21\times24\times80)=?$

21. A gentleman purchased 24 pounds of coffee, at 28c. a pound, and gave in payment 8 packages of sugar, each containing 7 pounds. What is the price per pound?

22. Bought 12 car loads of coal, each containing 32 tons, worth \$6 a ton, and gave in payment 8 boat loads of flour, each containing 36 barrels. What is the cost per barrel?

23. A grocer exchanged 54 firkins of butter, each containing 48 pounds, at 25c. per pound, for 12 chests of tea, each containing 60 pounds. What is the price per pound of the tea?

24. A company of wood-choppers cut 18 piles of wood of 36 cords each, and exchanged it for 12 car loads of coal of 42 tons each at $9 a ton. What is the price per cord of the wood?

25. A farmer planted 3 fields of corn of 120 rows, each row containing 72 hills, and each hill 5 grains. How many fields could he have planted with the same corn, each field having 96 rows of 75 hills each, and each hill containing 6 grains?

COMMON DIVISORS.

104. A ***Divisor*** of a number is any integer that will divide the number without a remainder.

Thus, 5 is a divisor of 10 or 15.

Since 10 and 15 are both divisible by 5, 5 is called a *common divisor* of 10 and 15. Hence,

105. A ***Common Divisor*** of several numbers is a divisor of each of them.

Since 5 is the greatest divisor of 10 and 15, it is called the *greatest common* divisor of those numbers. Hence,

106. The ***Greatest Common Divisor*** of several numbers is the greatest divisor of each of them.

ORAL.

1. What integers are divisors of 6? 8? 10? 12?
2. What integers are divisors of 9? 14? 15? 16?
3. What numbers are divisors of 18? 24? 56? 72?
4. What integers are divisors of 18 and 30? 24 and 36?
5. What numbers are divisors of 49 and 56? 63 and 72?
6. What numbers are divisors of 81 and 108? 84 and 96?

WRITTEN.

TO FIND THE GREATEST COMMON DIVISOR OF SEVERAL NUMBERS.

7. Find the greatest common divisor of 27, 36, and 45.

PROCESS, BY FACTORING.

$27 = 3 \times 3 \times 3$
$36 = 3 \times 3 \times 2 \times 2$
$45 = 3 \times 3 \times 5$

ANALYSIS.—Separating the numbers into their prime factors, it is found that the prime factors of 27 are 3, 3, and 3; the prime factors of 36 are 3, 3, 2, and 2; and the prime factors of 45 are 3, 3, and 5. Upon inspection it appears that the only prime factors found in each of these numbers are 3 and 3. Therefore, $3 \times 3 = 9$ is the greatest divisor of each of the numbers, and is therefore their greatest common divisor. Hence the following

Rule.—*Resolve the numbers into their prime factors, and find the product of the factors which are common.*

PRINCIPLES.

1. A divisor of a number is a divisor of any product of that number by integers.

2. The greatest common divisor of several numbers is either the least of the numbers or a factor of the least.

Find the greatest common divisor of the following numbers:

8. 24, 48, 64.
9. 36, 54, 72.
10. 21, 42, 84.
11. 48, 64, 80.
12. 45, 60, 75.
13. 18, 36, 90.
14. 24, 72, 96.
15. 96, 112, 144.
16. 90, 126, 162.
17. 60, 120, 180.
18. 108, 117, 126.
19. 132, 144, 156.
20. 105, 135, 180.
21. 128, 160, 208.
22. 180, 240, 400.
23. 270, 360, 540.
24. 280, 350, 420.
25. 288, 396, 432.
26. 384, 528, 576.
27. 576, 648, 864.
28. 76, 304, 380, and 456.
29. 84, 336, 420, and 504.
30. 92, 368, 460, and 552.
31. 96, 384, 480, and 576.
32. 108, 432, 540, and 648.
33. 600 ft., 700 ft., and 850 ft.
34. \$720, \$900, and \$1080.
35. 864c., 1080c., and 1296c.
36. 672 in., 840 in., and 1008 in.
37. 125 A., 375 A., and 500 A.
38. \$132, \$396, \$528, and \$660.
39. \$144, \$432, \$576, and \$720.

MULTIPLES.

The product of 4 times 5 is 20. The product 20 is therefore called a multiple of 4 or 5. Hence,

107. A ***Multiple*** of a number is the product obtained by using that number as one of its factors.

Factors and multiples are the reverse of each other, as in the above illustration, 4 is a factor of 20, and 20 is a multiple of 4.

In the expressions $3=3$, $3\times3=9$, and $3\times3\times3=27$, it is observed that 3, 9, and 27 are respectively the products of one, two, or three factors, each equal to 3, and they are therefore called powers of 3. Hence,

108. A ***Power*** of a number is the number itself, or is the product of several factors, each equal to the given number.

109. These powers are designated as *first*, *second*, *third*, etc., powers, according to the number of times the given number is used as a factor.

110. The ***First Power*** of a number is the number itself.

111. The ***Second Power*** of a number is the product of the number used twice as a factor.

The second power is usually called the ***Square.***

Thus, 16 is the square of 4, since it is the product of 4×4.

112. The ***Third Power*** of a number is the product of the number used three times as a factor.

The third power is usually called the ***Cube.***

Thus, 27 is the cube of 3, since it is the product of $3\times3\times3$.

113. The ***Exponent*** of a power is a number written to the right, and a little above the number, and shows the number of times the number is used as a factor.

Thus, in the expression 4^2, which is read 4 *square*$=4\times4=16$, 2 is the exponent, and shows that 4 is used twice as a factor. 4^3 which is read 4 *cube*, $=4\times4\times4=64$, and the exponent 3 shows that 4 is used as a factor 3 times.

114. The method of obtaining powers, though iden-

tical with all operations of *multiplication*, is called ***Involution.***

ORAL EXERCISES.

1. Name a number of which 3 is a factor; 4; 5; 6; 8.
2. Name a number of which 9 is a factor; 10; 12.
3. What is a multiple of 7? 11? 20? 30? 25?
4. Name a number of which 2 is an equal factor; 3; 4.
5. Name a number of which 5 is an equal factor; 6; 7.
6. Name a number of which 8 is an equal factor; 9; 10.
7. What is the first power of 4? 6? 8? 12? 100?
8. What is the square of 5? 7? 9? 11? 12?
9. What is the cube of 2? 3? 4? 5? 6?
10. What is the value of 3^2? 4^2? 6^2? 8^2? 9^2?
11. What is a factor of 12? 15? 24? 36? 40?
12. Name a factor of 21; 27; 32; 42; 45.
13. What is an equal factor of 9? 16? 25? 36? 49?
14. What is the first root of 7? 8? 15? 48? 72?
15. What is the square root of 16? 49? 64? 81? 121?
16. What is the cube root of 8? 27? 64? 125? 216?
17. What is the value of 4^2 plus the square root of 25?
18. What is the value of 5^3 plus the cube of 2?

COMMON MULTIPLES.

Since 12 is a multiple of both 3 and 4, it is called a *common multiple* of those numbers. Hence,

115. A ***Common Multiple*** of several numbers is a multiple of each of them.

Again, since there is no number less than 12 which is a multiple of both 3 and 4, it is therefore called the *least common multiple* of those numbers. Hence,

116. The ***Least Common Multiple*** of several numbers is the least number that is a multiple of each of them.

117. To find the Least Common Multiple of several numbers.

1. Find the least common multiple of 9, 10 and 12.

Process.

$9=3\times3$

$10=2\times5$

$12=2\times2\times3.$

L. C. M. is $2\times2\times3\times5\times3=180.$

Analysis.—Since the number sought is a multiple of the numbers 9, 10 and 12, it should be large enough to contain all the factors, of each of them, and since it is the least multiple of these numbers, it should be only large enough to contain the factors of each of them and no other factors. The least common multiple of 9, 10 and 12, will contain 12, or all the factors of 12, which are 2, 2 and 3. Since it will contain 10, it will contain all the factors of 10, which are 2 and 5; but the factor 2, found twice in 12, includes the factor 2 found once in 10: hence the 2 in 10 is rejected, and the other factor, 5 in 10, is retained as an additional factor of the least common multiple, so that it will contain all the factors of 10. Since the least common multiple will contain 9, it will contain all the factors of 9, which are 3 and 3; but the factor 3 found in 12 includes one factor 3 found in 9; hence, one factor 3 in 9 is rejected, and the other factor 3 in 9 is retained as an additional factor of the least common multiple, so that it will contain all the factors of 9. Therefore, all the factors of the least common multiple are 2, 2, 3, 5 and 3, and their product, which is 180, is the *least common multiple.* Hence the following

Rule.—I. *Separate the numbers into their prime factors.*

II. *Find the product of all the factors of the largest number, and such factors of all other numbers as are not found in the largest number.*

PRINCIPLES.

1. Every multiple of a number contains all its prime factors.

2. A common multiple of several numbers contains all the prime factors of each of them.

3. The least common multiple of several numbers contains all their prime factors, and no others.

4. The least common multiple of several numbers contains each of their prime factors the greatest number of times that it appears in either number.

Find the least common multiple of the following numbers.

2.	12.	16.	24.	36.	*16.*	36.	72.	108.	144.
3.	16.	24.	30.	40.	*17.*	40.	80.	120.	160.
4.	18.	27.	36.	45.	*18.*	45.	60.	90.	135.
5.	20.	25.	30.	35.	*19.*	48.	72.	96.	120.
6.	24.	36.	48.	54.	*20.*	56.	84.	112.	140.
7.	32.	40.	48.	56.	*21.*	54.	81.	108.	135.
8.	36.	45.	54.	63.	*22.*	60.	90.	150.	180.
9.	40.	50.	60.	75.	*23.*	64.	96.	128.	160.
10.	48.	60.	72.	84.	*24.*	72.	96.	144.	168.
11.	45.	60.	75.	90.	*25.*	72.	84.	108.	120.
12.	36.	54.	72.	90.	*26.*	75.	100.	125.	150.
13.	40.	60.	80.	100.	*27.*	80.	120.	160.	200.
14.	25.	50.	75.	100.	*28.*	96.	108.	120.	132.
15.	30.	60.	90.	120.	*29.*	96.	120.	132.	156.

PROBLEMS.

30. What is the least distance that can be exactly measured by either a yard measure, a bar 8 feet in length, or a pole 16 feet in length?

31. What is the size of the smallest tract of land that can be divided into lots of either 10 acres, 12 acres, 15 acres, or 18 acres? How many lots will there be of each?

32. What is the capacity of the smallest vessel that can be exactly filled by either an 8 gallon keg, a 21 gallon vessel, a 40 gallon cask, or a 63 gallon hogshead?

33. What is the smallest sum of money that can be used to purchase sheep at $9 a head, hogs at $15 a head, cows at $45 a head, or horses at $90 a head? How many of each may be bought?

34. What is the least sum of money that may be used

to buy land at $24 an acre, $60 an acre, wagons at $108 each, or suburban lots at $240 each?

35. A, B, C, and D, start together at a certain point to travel around a certain island. A can pass around it in 12 hours; B in 15 hours; C in 18 hours, and D in 24 hours. In how many hours will they all be together again? How many times will each one pass around the island before coming together?

REVIEW QUESTIONS.

What is a *factor* of a number? By what other names is a factor called? Define *root* of a number. How is the root of a number designated? What is the first root of a number? The second root? By what name is the second root usually known? What is the third root of a number? By what name is it usually known? How are numbers considered with reference to their factors? Define *prime* number. Define *composite* number. What is a *prime factor* of a number? *Composite factor?* What is a *common factor* of several numbers? What numbers are prime to each other? When is one number divisible by another? What are *even* numbers? *Odd* numbers? Define *factoring*. What is the first principle applicable to factoring? Second principle? Third principle? Fourth principle? How is a number resolved into its prime factors?

What is *cancellation?* Name the first principle upon which operations in cancellation are dependent. Second principle. Third principle. Give the *rule* for cancellation.

What is a *divisor* of a number? Define *common divisor* of several numbers. Define *greatest common divisor* of several numbers. Give the *rule* for finding the greatest common divisor of several numbers. What is the first principle applicable to the finding of the greatest common divisor? Second principle?

Define *multiple* of a number. What is a *power* of a number? How are powers designated? Define first power of a number. Second power. By what name is the second power usually known? Define third power of a number. By what name is it usually known? Define *exponent* of a power. Define *involution*. What is a *common multiple* of several numbers? Define *least common multiple* of several numbers. Give *rule* for finding the least common multiple. What is the first principle applicable to the finding of the least common multiple? Second principle? Third principle? Fourth principle?

SECTION IX.

FRACTIONS.

Art. 118. When any thing is divided into *two* equal parts, each part is called *one half.*

There are two pints in one quart; one pint is therefore *one half* of one quart. One half of two apples is one apple. One half of 100 apples is 50 apples.

When any thing is divided into *three* equal parts, each part is called *one third.* Two of the parts are called *two thirds.*

There are three feet in one yard. One foot is therefore *one third* of one yard; two feet are *two thirds* of one yard.

There are two halves in one thing; three thirds; four fourths; five fifths; etc.

119. A Fraction is one or more of the equal parts of a unit, or of any number regarded as a unit.

If an orange is divided equally among six boys, the part which each boy receives is called *a fraction. Two* of the parts are also called *a fraction;* so are three of them, or more.

120. The number or object which is divided into equal parts is called the ***Unit of the Fraction.***

Each of the equal parts into which the unit of the fraction is divided is called the ***Fractional Unit.***

In the illustration given above (Art. 119), the *orange* is the *unit of the fraction; one sixth of the orange* is the *fractional unit.*

121. Fractions are represented by figures, as follows:

One half	is written	$\frac{1}{2}$.	Two halves	are written	$\frac{2}{2}$.
One third	" "	$\frac{1}{3}$.	Two thirds	" "	$\frac{2}{3}$.
One fourth	" "	$\frac{1}{4}$.	Three fourths	" "	$\frac{3}{4}$.
One eighth	" "	$\frac{1}{8}$.	Seven eighths	" "	$\frac{7}{8}$.
One sixtieth	" "	$\frac{1}{60}$.	Forty sixtieths	"	$\frac{40}{60}$.

122. The numbers, or ***Terms,*** which represent fractions, are called ***Numerator*** and ***Denominator.***

123. The numerator is written *above* a short horizontal or oblique line, and the denominator *below* it. $\frac{1}{2}$, $\frac{3}{5}$, $2/3$.

124. The ***Denominator*** shows the number of equal parts into which the unit of the fraction is divided.

125. The ***Numerator*** shows either the unit of the fraction, or the number of fractional units in the fraction.

When *one* orange is divided into *six* equal parts, each part is represented by the fraction $\frac{1}{6}$, in which 6 is the *denominator*, and shows into how many equal parts the orange has been divided. The *numerator* 1 shows that *one* orange has been divided.

The fraction $\frac{2}{6}$ represents *two* of the *six* equal parts into which *one* orange has been divided; or, it represents *one sixth* part of *two* oranges. The pupil can readily see that *one sixth* of two oranges is equal to *two sixths* of one orange.

126. A ***Mixed Number*** is composed of an integer and a fraction. $1\frac{1}{2}$, $25\frac{4}{5}$.

Read:

1. $\frac{1}{2}$.	*6.* $\frac{7}{10}$.	*11.* $\frac{21}{100}$.	*16.* $5\frac{2}{3}$.
2. $\frac{3}{5}$.	*7.* $\frac{14}{15}$.	*12.* $\frac{5}{125}$.	*17.* $17\frac{3}{8}$.
3. $\frac{3}{8}$.	*8.* $\frac{8}{13}$.	*13.* $1\frac{25}{5}$.	*18.* $24\frac{4}{9}$.
4. $\frac{6}{7}$.	*9.* $\frac{4}{20}$.	*14.* $\frac{24}{168}$.	*19.* $181\frac{2}{5}$.
5. $\frac{5}{9}$.	*10.* $\frac{15}{60}$.	*15.* $\frac{140}{10}$.	*20.* $290\frac{8}{15}$.

Write in figures:

21. Four tenths.	*26.* Sixteen fortieths.
22. Six ninths.	*27.* Twenty five-hundredths.
23. Ten twelfths.	*28.* Thirty-six eighty-fifths.
24. Eight elevenths.	*29.* Three and three eighths.
25. Two sevenths.	*30.* Sixteen and five ninths.

127. If three oranges are each divided into six equal parts, or *sixths*, there are *eighteen sixths*, $\frac{18}{6}$. The number of *oranges* in these *eighteen sixths*, $\frac{18}{6}$, can be found by dividing the numerator by the denominator, $18 \div 6 = 3$. So, $\frac{24}{6} = 24 \div 6 = 4$; $\frac{24}{8} = 24 \div 8 = 3$; etc.

The quotient resulting from the division of the numerator by the denominator is the ***Value of the Fraction.***

128. Whenever the numerator equals the denominator, the value of the fraction is *one*. $\frac{4}{4} = 1$.

Whenever the numerator exceeds the denominator, the value of the fraction is greater than *one*. $\frac{12}{4} = 3$.

Whenever the numerator is less than the denominator, the value of the fraction is less than *one*. $\frac{1}{3}$ is less than 1. $\frac{14}{15}$ is less than 1.

129. A ***Proper Fraction*** is a fraction whose value is less than *one*. $\frac{1}{8}$, $\frac{11}{12}$.

130. An ***Improper Fraction*** is a fraction whose value is equal to, or greater than, *one*. $\frac{6}{6}$, $\frac{5}{4}$.

REVIEW QUESTIONS.

What is a fraction? What are the terms of a fraction? Define each term. What is a mixed number? How is the value of a fraction found? What is the difference between a proper fraction and an improper fraction? Define each.

REDUCTION OF FRACTIONS.

CASE I.—To reduce an integer or a mixed number to an improper fraction.

ORAL.

131. *1.* How many half oranges in 3 oranges?

SOLUTION.—In 3 oranges there are 3 times as many half oranges as in 1 orange. In one orange there are 2 half oranges; in 3 oranges there are three times 2 half oranges, or 6 half oranges.

2. How many thirds of an orange in 3 oranges?
3. How many fourths of an apple in 4 apples.
4. How many fifths of a dollar in ten dollars?
5. In 5 how many halves? Thirds? Fourths?
6. In 8 how many fifths? Sixths? Ninths?
7. Change 6 to 10ths. To 12ths.
8. Reduce 9 to fourths. To sevenths.
9. Reduce $3\frac{3}{4}$ to fourths.

SOLUTION.— $3\frac{3}{4}=3+\frac{3}{4}$. In 3 units there are 3 times as many fourths as in 1 unit. In one unit there are 4 fourths; in 3 units there are 3 times 4 fourths, or 12 fourths. 12 fourths + 3 fourths $=\frac{15}{4}$.

Rule.—*Find the product of the integer and the denominator. To this product add the numerator of the fraction, if there be any. The result is the numerator of the required fraction.*

NOTE.—It will be observed that the rule directs the reduction of the integer to the same denominator as the fraction, previous to their addition. This is in accordance with the principle (Art. 33) that like numbers only can be added.

10. Reduce $1\frac{1}{2}$ to halves.
11. Reduce $2\frac{2}{3}$ to 3rds.
12. Reduce $5\frac{4}{5}$ to 5ths.
13. Reduce $6\frac{3}{4}$ to 4ths.
14. Reduce $8\frac{5}{6}$ to 6ths.
15. Reduce $7\frac{7}{10}$ to 10ths.
16. Reduce $9\frac{4}{9}$ to 9ths.
17. Reduce $10\frac{2}{7}$ to 7ths.

WRITTEN.

Reduce to improper fractions:

18. $29\frac{4}{15}$.
19. $40\frac{7}{18}$.
20. $56\frac{22}{41}$.
21. $37\frac{5}{19}$.
22. $89\frac{6}{11}$.
23. $75\frac{20}{39}$.
24. $84\frac{24}{11}$.
25. $698\frac{3}{90}$.
26. $8191\frac{6}{17}$.
27. $925\frac{31}{32}$.
28. $740\frac{5}{61}$.
29. $8371\frac{2}{75}$.

CASE II.—To reduce an improper fraction to an integer or a mixed number.

ORAL.

132. *1.* In 24 sixths of an orange, how many oranges?

SOLUTION.—There are six sixths of an orange in one orange. In 24 sixths of an orange there are as many oranges as 6 sixths are con-

tained times in 24 sixths, which are 4 times. Hence, in 24 sixths of an orange there are 4 oranges.

2. In $\frac{13}{3}$ how many units?

SOLUTION. — There are 3 thirds in one unit. In $\frac{13}{3}$ there are as many units as $\frac{3}{3}$ are contained times in $\frac{13}{3}$, which are $4\frac{1}{3}$ times. Hence, $\frac{13}{3}=4\frac{1}{3}$.

Rule. — *Divide the numerator of the fraction by its denominator*

Reduce to integers or mixed numbers:

3. $\frac{18}{6}$.	*7.* $\frac{36}{9}$.	*11.* $\frac{60}{8}$.	*15.* $\frac{80}{9}$.
4. $\frac{20}{5}$.	*8.* $\frac{35}{7}$.	*12* $\frac{39}{9}$.	*16.* $\frac{39}{5}$.
5. $\frac{48}{8}$.	*9.* $\frac{40}{8}$.	*13.* $\frac{50}{6}$.	*17.* $\frac{63}{12}$.
6. $\frac{60}{10}$.	*10.* $\frac{56}{7}$.	*14.* $\frac{73}{8}$.	*18.* $\frac{41}{9}$.

WRITTEN.

19. $\frac{109}{10}$.	*23.* $\frac{281}{16}$.	*27.* $\frac{542}{37}$.
20. $\frac{249}{7}$.	*24.* $\frac{329}{18}$.	*28.* $\frac{409}{50}$.
21. $\frac{325}{25}$.	*25.* $\frac{421}{40}$.	*29.* $\frac{625}{25}$.
22. $\frac{450}{30}$.	*26.* $\frac{673}{29}$.	*30.* $\frac{440}{40}$.

CASE III. — To reduce a fraction to higher or lower terms.

133. In one orange there are four fourths; therefore, in one half of an orange there is one half of four fourths, or two fourths; that is, $\frac{1}{2}=\frac{2}{4}$. In one orange there are ten tenths; therefore, in one half of an orange there is one half of ten tenths, or five tenths; that is, $\frac{1}{2}=\frac{5}{10}$. These fractions, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{5}{10}$ have the same value; each is equal to one half of the same unit. They are, therefore, called *equivalent fractions.*

134. Equivalent Fractions are fractions of different expression, but of the same value.

135. The value of a fraction is the quotient resulting from the division of the numerator by the denominator. (Art. 127.) That is,

The Numerator is the Dividend.

The Denominator is the Divisor.

The Value of the Fraction is the Quotient.

Therefore, the principles of division apply to the terms of a fraction, and both terms of a fraction may be multiplied or divided by the same number, without changing the value of the fraction. One half may be changed to two fourths, or three sixths, or ten twentieths, by multiplying both terms of the fraction $\frac{1}{2}$, by 2, by 3, or by 10. So, two fourths, three sixths, and ten twentieths, may be changed to one half, by dividing both terms by 2, by 3, or by 10.

ORAL.

1. Change $\frac{3}{5}$ of an orange to 10ths.

Solution.—*One fifth* is equal to *two tenths*. Then 3 fifths are equal to 3 times 2 tenths, which are 6 tenths. Hence, $\frac{3}{5} = \frac{6}{10}$.

Rule.—*Multiply or divide each term of the fraction by the number necessary to change the given denominator to the required denominator.*

2. Reduce $\frac{1}{2}$ to 8ths.
3. Reduce $\frac{3}{4}$ to 8ths.
4. Reduce $\frac{5}{7}$ to 14ths.
5. Reduce $\frac{2}{3}$ to 24ths.
6. Reduce $\frac{4}{5}$ to 20ths.
7. Reduce $\frac{3}{8}$ to 24ths.
8. Reduce $\frac{9}{10}$ to 40ths.
9. Reduce $\frac{4}{9}$ to 36ths.

10. Reduce $\frac{6}{8}$ to 4ths.
11. Reduce $\frac{5}{15}$ to 3rds.
12. Reduce $\frac{4}{20}$ to 5ths.
13. Reduce $\frac{10}{12}$ to 6ths.
14. Reduce $\frac{8}{24}$ to 3rds.
15. Reduce $\frac{2}{24}$ to 4ths.
16. Reduce $\frac{20}{30}$ to 3rds.
17. Reduce $\frac{25}{40}$ to 8ths.

WRITTEN.

18. Reduce $\frac{6}{72}$ to 84ths.
19. Reduce $\frac{5}{20}$ to 100ths.
20. Reduce $\frac{40}{60}$ to 240ths.
21. Reduce $\frac{8}{96}$ to 288ths.
22. Reduce $\frac{50}{150}$ to 3rds.
23. Reduce $\frac{40}{160}$ to 4ths.
24. Reduce $\frac{100}{200}$ to 5ths.
25. Reduce $\frac{15}{75}$ to 5ths.

CASE IV.—To reduce a fraction to lowest terms.

136. A fraction is in its ***Lowest Terms*** when its terms are prime to each other. $\frac{3}{4}$, $\frac{15}{16}$.

ORAL.

1. Reduce $\frac{18}{24}$ to lowest terms.

ANALYSIS.—Since (Art. 135) the value of a fraction is not changed by the division of both terms by the same number, each term of the fraction $\frac{18}{24}$ may be divided by any factor common to both; and this division continued until the terms of the resulting fraction are prime to each other.

SOLUTION I.—Divide each term first by 2. $\frac{18 \div 2}{24 \div 2} = \frac{9}{12}$. Then divide each term by 3. $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$. Since 3 and 4 are prime to each other, the fraction $\frac{18}{24}$, reduced to lowest terms, is $\frac{3}{4}$.

SOLUTION II.—Divide each term by the greatest common divisor of the terms. The G. C. D. of 18 and 24 is 6. $\frac{18 \div 6}{24 \div 6} = \frac{3}{4}$

Rule I.—*Divide each term by any factor common to both terms; so continue until the terms are prime to each other.*

Rule II.—*Divide both terms by their greatest common divisor.*

Reduce to lowest terms.

2. $\frac{3}{6}$; $\frac{4}{8}$; $\frac{5}{15}$.

3. $\frac{4}{16}$; $\frac{20}{25}$; $\frac{12}{24}$.

4. $\frac{20}{30}$; $\frac{18}{27}$; $\frac{12}{48}$.

5. $\frac{32}{36}$; $\frac{40}{45}$; $\frac{45}{60}$.

6. $\frac{8}{64}$; $\frac{15}{35}$; $\frac{25}{75}$.

7. $\frac{24}{60}$; $\frac{18}{72}$; $\frac{60}{84}$.

8. $\frac{33}{44}$; $\frac{72}{81}$; $\frac{105}{144}$.

9. $\frac{35}{42}$; $\frac{36}{56}$; $\frac{24}{60}$.

WRITTEN.

10. $\frac{72}{124}$; $\frac{30}{156}$; $\frac{28}{140}$.

11. $\frac{56}{168}$; $\frac{20}{120}$; $\frac{85}{170}$.

12. $\frac{12}{156}$; $\frac{22}{110}$; $\frac{41}{123}$.

13. $\frac{60}{180}$; $\frac{32}{160}$; $\frac{39}{78}$.

14. $\frac{45}{180}$; $\frac{27}{81}$; $\frac{91}{182}$.

15. $\frac{125}{500}$; $\frac{80}{640}$; $\frac{21}{105}$.

16. $\frac{90}{270}$; $\frac{18}{540}$; $\frac{23}{138}$.

17. $\frac{81}{189}$; $\frac{32}{128}$; $\frac{42}{168}$.

CASE V.—To reduce several fractions to equivalent fractions having a common denominator.

137. Fractions having equal denominators are said to have a ***Common Denominator.***

ORAL.

1. Change the fractions $\frac{1}{2}$ and $\frac{1}{4}$ to equivalent fractions having a common denominator.

SOLUTION I.—Reduce $\frac{1}{2}$ to 4ths. $\frac{1}{2}$ equals $\frac{2}{4}$. The required fractions are $\frac{2}{4}$ and $\frac{1}{4}$.

SOLUTION II.—Reduce to 8ths. $\frac{1}{2}=\frac{4}{8}$; $\frac{1}{4}=\frac{2}{8}$. The required fractions are $\frac{4}{8}$ and $\frac{2}{8}$.

SOLUTION III.—Reduce to 12ths. $\frac{1}{2}=\frac{6}{12}$; $\frac{1}{4}=\frac{3}{12}$. The required fractions are $\frac{6}{12}$ and $\frac{3}{12}$.

It is evident that *any common multiple of the denominators* of the given fractions may be taken as the *common denominator.*

2. Change $\frac{5}{20}$ and $\frac{2}{15}$ to equivalent fractions having a common denominator.

SOLUTION.—Reduce to lowest terms. $\frac{5}{20}=\frac{1}{4}$; $\frac{2}{15}=\frac{4}{5}$. Take any multiple of 4 and 5, say 20. By case III, $\frac{1}{4}=\frac{5}{20}$; $\frac{4}{5}=\frac{16}{20}$.

Reduce to common denominator.

2. $\frac{1}{2}$ and $\frac{1}{3}$.	*5.* $\frac{2}{3}$ and $\frac{4}{6}$.	*7.* $\frac{3}{15}$ and $\frac{3}{10}$.
3. $\frac{2}{5}$ and $\frac{5}{6}$.	*6.* $\frac{3}{8}$ and $\frac{1}{2}$.	*8.* $\frac{4}{7}$ and $\frac{3}{4}$.

CASE VI.—To reduce several fractions to equivalent fractions having the Least Common Denominator.

138. *1.* Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to equivalent fractions having the least common denominator.

ANALYSIS.—Any multiple of the denominators 2 and 3 may be taken for a *common* denominator; but it is evident that the *least common* denominator of the fractions is the *least common multiple* of the denominators

SOLUTION.—The least common multiple of the denominators, 2 and 3, is 6 By Case III, $\frac{1}{2}=\frac{3}{6}$; $\frac{1}{3}=\frac{2}{6}$.

2 Reduce $\frac{2}{5}$, $\frac{5}{6}$, and $\frac{3}{8}$, to equivalent fractions having the least common denominator.

SOLUTION.—The least common multiple of the denominators 5, 6, and 8, is 120.

$1=\frac{120}{120}$; $\frac{1}{5}=\frac{1}{5}$ of $\frac{120}{120}=\frac{24}{120}$; $\frac{2}{5}=\frac{2}{5}$ of $\frac{120}{120}=\frac{48}{120}$.

$1=\frac{120}{120}$; $\frac{1}{6}=\frac{1}{6}$ of $\frac{120}{120}=\frac{20}{120}$; $\frac{5}{6}=\frac{5}{6}$ of $\frac{120}{120}=\frac{100}{120}$.

$1=\frac{120}{120}$; $\frac{1}{8}=\frac{1}{8}$ of $\frac{120}{120}=\frac{15}{120}$; $\frac{3}{8}=\frac{3}{8}$ of $\frac{120}{120}=\frac{45}{120}$.

Rule.—*Find the least common multiple of the denominators. This is the least common denominator of the*

fractions. Multiply each term of each fraction by the quotient of the least common denominator divided by the denominator of the fraction.

Reduce to equivalent fractions having the least common denominator:

ORAL.

3. $\frac{1}{4}$ and $\frac{2}{5}$.
4. $\frac{3}{8}$ and $\frac{5}{6}$.
5. $\frac{2}{3}$ and $\frac{5}{9}$.
6. $\frac{3}{7}$ and $\frac{2}{9}$.
7. $\frac{4}{5}$ and $\frac{1}{12}$.
8. $\frac{2}{7}$ and $\frac{5}{11}$.
9. $\frac{5}{6}$ and $\frac{4}{9}$.
10. $\frac{2}{3}$ and $\frac{7}{12}$.
11. $\frac{3}{8}$ and $\frac{3}{10}$.
12. $\frac{7}{11}$ and $\frac{5}{12}$.
13. $\frac{2}{9}$ and $\frac{7}{10}$.
14. $\frac{3}{4}$ and $\frac{11}{12}$.

WRITTEN.

15. $\frac{3}{8}$, $\frac{4}{5}$ and $\frac{3}{4}$.
16. $\frac{9}{10}$, $\frac{5}{16}$ and $\frac{7}{12}$.
17. $\frac{20}{24}$, $\frac{2}{6}$ and $\frac{7}{8}$.
18. $\frac{7}{49}$, $\frac{6}{42}$ and $\frac{4}{28}$.
19. $\frac{4}{5}$, $\frac{7}{20}$ and $\frac{7}{10}$.
20. $\frac{2}{6}$, $\frac{3}{8}$ and $\frac{5}{6}$.
21. $\frac{4}{9}$, $\frac{3}{7}$ and $\frac{1}{2}$.
22. $\frac{2}{3}$, $\frac{5}{9}$ and $\frac{8}{12}$.
23. $\frac{4}{20}$, $\frac{5}{12}$ and $\frac{2}{9}$.
24. $\frac{3}{11}$, $\frac{2}{7}$ and $\frac{2}{3}$.

REVIEW QUESTIONS.

How may an integer be reduced to a fraction? Will such a fraction be proper or improper? Why? How may a mixed number be reduced to an improper fraction? How may an improper fraction be changed to an integer or mixed number? When will the result be an integer? When a mixed number? How may a fraction be reduced to lower terms? To higher? What are the lowest terms of a fraction?

Define equivalent fractions. To what terms in division do the terms of a fraction correspond? What principles of division apply to fractions? How are fractions reduced to lowest terms? When do fractions have a common denominator? How are fractions reduced to equivalent fractions having a common denominator? Name several purposes for which fractions are thus reduced. How are fractions reduced to equivalent fractions having the least common denominator?

ADDITION OF FRACTIONS.

ORAL.

139. *1.* William has $\frac{4}{5}$ of a dollar, and Henry has $\frac{3}{5}$ of a dollar. How many fifths of a dollar have both?

SOLUTION.—They both have the sum of 4 fifths of a dollar and 3 fifths of a dollar, which is $\frac{7}{5}$ of a dollar, equal to 1 dollar and $\frac{2}{5}$ of a dollar

2. Mary learned her spelling lesson in $\frac{5}{8}$ of an hour, and her reading lesson in $\frac{7}{8}$ of an hour. In what time did she learn both lessons?

3. Find the sum of $\frac{5}{12}$, $\frac{3}{12}$ and $\frac{7}{12}$.

4. Mr. Adams bought $\frac{2}{3}$ of a ton of coal, and Mr. Bates bought $\frac{3}{4}$ of a ton of coal. How much coal did they both buy?

ANALYSIS.—Since $\frac{2}{3}$ and $\frac{3}{4}$ have not the same fractional unit, that is, since $\frac{1}{3}$ is not equal to $\frac{1}{4}$, the sum is neither 5 *thirds* nor 5 *fourths*. But $\frac{2}{3}$ and $\frac{3}{4}$ may be changed to equivalent fractions whose denominators are 12, 24, 36, or any other multiple of 3 and 4.

SOLUTION.— $\frac{2}{3}=\frac{8}{12}$; $\frac{3}{4}=\frac{9}{12}$; $\frac{8}{12}+\frac{9}{12}=1\frac{7}{12}=1\frac{5}{12}$. They bought the sum of $\frac{2}{3}$ of a ton and $\frac{3}{4}$ of a ton, which is $1\frac{5}{12}$ tons.

Rule.—*Reduce the fractions to equivalent fractions having a common denominator; find the sum of the numerators; divide this sum by the common denominator.*

NOTE.—It is generally best to reduce the fractions to their *least* common denominator.

5. Find sum of $\frac{3}{4}$ and $\frac{2}{5}$.

6. Find sum of $\frac{5}{6}$ and $\frac{1}{4}$.

7. Find sum of $\frac{2}{3}$ and $\frac{5}{9}$.

NOTE.—Reduce $\frac{5}{9}$ to 9ths.

8. James spent $\frac{3}{4}$ of a dollar on Monday, $\frac{1}{2}$ of a dollar on Tuesday, and $\frac{3}{8}$ of a dollar on Wednesday. How much did he spend in the three days?

9. $\frac{3}{7}+\frac{1}{3}=?$

10. $\frac{4}{5}+\frac{3}{5}=?$

11. $\frac{5}{6}+\frac{2}{11}=?$

12. $\frac{7}{12}+\frac{7}{24}=?$

13. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=?$

14. $\frac{2}{3}+\frac{5}{6}+\frac{7}{8}=?$

15. $\frac{3}{5}+\frac{2}{3}+\frac{1}{6}=?$

16. $\frac{3}{8}+\frac{3}{4}+\frac{7}{10}=?$

17. Find sum of $2\frac{1}{2}$ and $5\frac{9}{10}$.

SUGGESTIONS.—Reduce the fractions to 10ths, and find their sum, which is $1\frac{2}{5}$. Add this to the sum of the integers 2 and 5.

18. $1\frac{3}{4}+4\frac{2}{5}=?$

19. $5\frac{4}{9}+3\frac{7}{12}=?$

20. $8\frac{3}{7}+5\frac{3}{4}=?$

21. $3\frac{1}{2}+4\frac{7}{12}=?$

WRITTEN.

22. $\frac{3}{10}+\frac{4}{5}=?$
23. $\frac{5}{12}+\frac{3}{11}=?$
24. $\frac{5}{9}+\frac{7}{12}=?$
25. $\frac{16}{24}+\frac{3}{8}=?$
26. $\frac{20}{21}+\frac{5}{7}=?$
27. $\frac{3}{4}+\frac{5}{8}+\frac{3}{4}=?$
28. $\frac{4}{5}+\frac{11}{12}+\frac{9}{10}=?$
29. $\frac{4}{5}+\frac{3}{4}+\frac{3}{5}=?$
30. $13\frac{2}{7}+10\frac{3}{4}+5\frac{2}{5}=?$
31. $29\frac{1}{2}+7\frac{2}{11}+18\frac{5}{7}=?$
32. $9\frac{2}{5}+3\frac{8}{9}+1\frac{2}{3}=?$
33. $4\frac{6}{7}+15\frac{1}{2}+60\frac{3}{5}=?$
34. $20\frac{2}{3}+11\frac{3}{4}+51\frac{3}{84}=?$
35. $7\frac{5}{24}+91\frac{9}{8}+4\frac{7}{12}=?$
36. $5\frac{3}{10}+16\frac{1}{12}+8\frac{5}{6}=?$
37. $7\frac{2}{7}+34\frac{1}{9}+5\frac{4}{63}=?$

38. A horse costs $\$147\frac{1}{2}$; the harness $\$65\frac{2}{3}$; the buggy $\$231\frac{3}{4}$. Find cost of all.

39. Mr. C's house is worth $\$5275\frac{3}{4}$; his furniture \$1300; his barn $\$784\frac{2}{3}$. What are they all worth?

40. One field contains $75\frac{5}{6}$ acres; a second field has $60\frac{1}{4}$ acres; a third, $109\frac{7}{12}$ acres. How many acres in all?

SUBTRACTION OF FRACTIONS.

ORAL.

140. *1.* Mr. D. owned $\frac{5}{9}$ of a mill, and sold $\frac{2}{9}$ of it; what part of the mill did he then own?

SOLUTION.—He owned the difference between $\frac{5}{9}$ and $\frac{2}{9}$ of the mill, which is $\frac{3}{9}$ of the mill, equal to $\frac{1}{3}$ of the mill.

2. Mr. E. owns $\frac{5}{12}$ of a ship, and Mr. F. owns $\frac{1}{12}$ of it. How much more does Mr. E. own than Mr. F.?

3. Find the difference between $\frac{12}{19}$ and $\frac{7}{19}$.

4. Find the difference between $\frac{5}{6}$ and $\frac{3}{5}$.

SOLUTION.—Reduce to equivalent fractions having least common denominator. $\frac{5}{6}=\frac{25}{30}$; $\frac{3}{5}=\frac{18}{30}$. $\frac{25}{30}-\frac{18}{30}=\frac{7}{30}$.

Rule.—*Reduce the fractions to equivalent fractions having a common denominator; find the difference of the numerators; divide this difference by the common denominator.*

5. $\frac{5}{8}-\frac{3}{4}=?$
6. $\frac{3}{7}-\frac{1}{14}=?$
7. $\frac{1}{2}-\frac{2}{5}=?$
8. $1\frac{1}{12}-\frac{4}{7}=$
9. $\frac{5}{9}-\frac{2}{7}=?$
10. $\frac{9}{11}-\frac{3}{4}=?$
11. $\frac{3}{5}-\frac{1}{12}=?$
12. $\frac{4}{7}-\frac{1}{8}=?$

13. Find difference of $8\frac{3}{4}$ and $5\frac{1}{4}$.

14. Subtract $15\frac{3}{7}$ from $20\frac{6}{7}$.

15. $8\frac{1}{2}-4\frac{3}{4}=?$

SOLUTION I.—The $\frac{1}{2}$ in the minuend is equal to $\frac{2}{4}$. $\frac{3}{4}$ can not be subtracted from $\frac{2}{4}$. Take 1 of the 8 units; reduce to 4ths, and add to the $\frac{2}{4}$. $\frac{4}{4}+\frac{2}{4}=\frac{6}{4}$. $7\frac{6}{4}-4\frac{3}{4}=3\frac{3}{4}$.

SOLUTION II.— $8-4\frac{3}{4}=3\frac{1}{4}$; $3\frac{1}{4}+\frac{1}{2}=3\frac{3}{4}$.

16. $5\frac{1}{2}-1\frac{2}{3}=?$ *18.* $7\frac{2}{9}-5\frac{5}{6}=?$

17. $3\frac{2}{5}-2\frac{3}{4}=?$ *19.* $6\frac{1}{3}-2\frac{3}{7}=?$

WRITTEN.

20. $17\frac{5}{6}-8\frac{3}{7}=?$ *22.* $32\frac{3}{5}-21\frac{4}{9}=?$

21. $25\frac{2}{11}-16\frac{5}{8}=?$ *23.* $18\frac{6}{7}-15\frac{1}{12}=?$

ADDITION AND SUBTRACTION.

24. $\frac{1}{2}+\frac{2}{5}-\frac{1}{4}=?$ *28.* $\frac{5}{9}+\frac{2}{3}-\frac{2}{27}=?$

25. $\frac{2}{3}+\frac{5}{8}-\frac{7}{12}=?$ *29.* $1\frac{1}{2}+2\frac{1}{5}-3\frac{7}{10}=?$

26. $\frac{5}{6}-\frac{1}{4}+\frac{3}{8}=?$ *30.* $3\frac{3}{8}+5\frac{1}{4}-2\frac{7}{16}=?$

27. $\frac{5}{7}-\frac{1}{2}+\frac{1}{14}=?$ *31.* $\frac{3}{5}+4\frac{2}{4}-\frac{3}{8}=?$

32. Mr. B. having \$50, paid one bill of $\$7\frac{2}{5}$, and another of $\$10\frac{2}{5}$. How much money had he remaining?

33. Mr. C. spends $\frac{3}{7}$ of his time at work, $\frac{1}{8}$ in study. What part of his time is devoted to other things?

34. $2\frac{2}{3}+5\frac{3}{8}$ are how much less than $8\frac{5}{12}$?

35. $40\frac{2}{9}$ is the sum of $2\frac{5}{6}$, $15\frac{3}{4}$, and what other number?

MULTIPLICATION OF FRACTIONS.

CASE I.—To multiply a fraction by an integer.

141. The principle in multiplication, that the product is of the same denomination as the multiplicand, is true in fractions as well as in integers. 5 times 3 *oranges* are 15 *oranges;* 5 times 3 *sheep* are 15 *sheep;* 5 times 3 *eighths* are

15 *eighths*. The *denominator* of the fraction is the *denomination*, or *kind*, of the multiplicand and product.

ORAL.

1. Multiply $\frac{3}{8}$ by 5.

SOLUTION.— 5 times 3 *eighths* are 15 eighths, $=1\frac{7}{8}$.

2. Multiply $\frac{2}{3}$ by 4.
3. Multiply $\frac{4}{9}$ by 2.
4. Multiply $\frac{5}{8}$ by 3.
5. Multiply $\frac{3}{10}$ by 7.

6. Multiply $\frac{5}{6}$ by 3.

ANALYSIS.—By the method used in solving example 1, $\frac{5}{6}\times 3=\frac{15}{6}=\frac{5}{2}=2\frac{1}{2}$. This method multiplies the fractional unit $\frac{1}{6}$; the product, $\frac{15}{6}$, contains 3 times as many fractional units, or parts, as the multiplicand, $\frac{5}{6}$. But the same result may be secured by making each fractional unit 3 times as large, instead of taking 3 times as many of the same size. Thus, 3 times $\frac{1}{6}$ is $\frac{1}{2}$; 3 times $\frac{5}{6}$ are $\frac{5}{2}$. Therefore, instead of *multiplying* the *numerator* of the fraction by the integer, the *denominator* of the fraction may be *divided* by the integer, where the integer is a factor of the denominator.

SOLUTION.— $\frac{5}{6}\times 3=\frac{5}{6\div 3}=\frac{5}{2}=2\frac{1}{2}$.

Rule.—*Multiply the numerator*, OR *divide the denominator, by the integer.*

7. Multiply $\frac{3}{4}$ by 2.
8. Multiply $\frac{5}{12}$ by 4.
9. Multiply $\frac{3}{8}$ by 8.
10. Multiply $\frac{7}{10}$ by 5.
11. Multiply $\frac{8}{21}$ by 7.
12. Multiply $\frac{3}{16}$ by 8.

13. Find the cost of 8 pounds of butter, at $\frac{1}{6}$ of a dollar per pound.

14. How many bushels of grain will a horse eat in 12 days, if he eat $\frac{5}{8}$ of a bushel in one day?

15. What is the cost of 10 yards of cloth, at $\frac{7}{8}$ of a dollar per yard?

16. Multiply $3\frac{2}{3}$ by 5.

SOLUTION.— 5 times 3=15; 5 times $\frac{2}{3}=\frac{10}{3}=3\frac{1}{3}$; $15+3\frac{1}{3}=18\frac{1}{3}$.

17. Multiply $2\frac{1}{2}$ by 3.
18. Multiply $6\frac{3}{4}$ by 2.
19. Multiply $5\frac{3}{7}$ by 6.
20. Multiply $8\frac{5}{9}$ by 7.

WRITTEN.

21. $\frac{5}{8}\times 25=?$
22. $\frac{11}{12}\times 21=?$
23. $\frac{9}{10}\times 33=?$
24. $\frac{7}{12}\times 29=?$
25. $\frac{5}{11}\times 40=?$
26. $\frac{3}{4}\times 25=?$

27. $\frac{10}{21}\times 14=?$

SUGGESTION.—Cancel factors common to the denominator and the integer.

28. $\frac{12}{25}\times 15=?$
29. $\frac{24}{40}\times 21=?$
30. $\frac{16}{33}\times 22=?$
31. $\frac{49}{50}\times 100=?$
32. $8\frac{5}{6}\times 15=?$
33. $9\frac{3}{5}\times 20=?$
34. $6\frac{10}{11}\times 33=?$
35. $5\frac{8}{9}\times 56=?$

36. Find cost of 24 acres of land at $\$12\frac{1}{2}$ per acre.

37. What is the value of 32 bales of cotton at $\$46\frac{3}{8}$ per bale?

38. Cost of 4 houses worth $\$3251\frac{3}{10}$ each?

39. How many miles can a locomotive run in 9 hours, at the rate of $36\frac{2}{3}$ miles an hour?

40. Cost of 12 horses, each valued at $\$210\frac{3}{5}$?

CASE II.—To multiply an integer by a fraction.

ORAL.

142. *1.* Multiply 8 by $\frac{3}{4}$.

ANALYSIS.—To multiply 8 by $\frac{3}{4}$ is to find $\frac{3}{4}$ of 8. One fourth of 8 is obtained by dividing 8 by 4; 3 fourths=3 times 1 fourth.

SOLUTION.— $\frac{1}{4}$ of 8 is 2; $\frac{3}{4}$ of 8=3 times 2,=6.

Special Rule.—*Take such a part of the integer as is indicated by the fractional unit; multiply this quotient by the numerator.*

2. Multiply 9 by $\frac{2}{3}$.
3. Multiply 12 by $\frac{5}{6}$.
4. Multiply 14 by $\frac{3}{7}$.
5. Multiply 25 by $\frac{4}{5}$.
6. Multiply 40 by $\frac{3}{8}$.
7. Multiply 60 by $\frac{5}{12}$.

8. Multiply 4 by $\frac{2}{3}$.

SOLUTION.— $\frac{1}{3}$ of 4 is $\frac{4}{3}$; $\frac{2}{3}$ of 4 are 2 times $\frac{4}{3}=\frac{8}{3}=2\frac{2}{3}$.

9. Multiply 5 by $\frac{3}{4}$.
10. Multiply 6 by $\frac{2}{5}$.
11. Multiply 8 by $\frac{3}{7}$.
12. Multiply 4 by $\frac{5}{6}$.
13. Multiply 10 by $\frac{3}{8}$.
14. Multiply 6 by $\frac{3}{5}$.

15. Multiply 4 by $\frac{3}{8}$.

NOTE.—When the integer is a factor of the denominator, proceed as in example 6, Case I. $4\times\frac{3}{8}=\frac{3}{8\div4}=\frac{3}{2}=1\frac{1}{2}$.

16. Multiply 5 by $\frac{4}{5}$.
17. Multiply 8 by $\frac{5}{16}$.
18. Multiply 6 by $\frac{5}{12}$.
19. Multiply 10 by $\frac{3}{20}$.

Since either factor may be regarded as the multiplier, Cases I and II, are identical in operation. $\frac{4}{5}\times7=5\frac{3}{5}$, and $7\times\frac{4}{5}=5\frac{3}{5}$.

Rule.—*Multiply the numerator*, OR *divide the denominator, by the integer.*

20. What is the cost of $\frac{3}{4}$ of a ton of coal, at \$9 per ton?
21. Find cost of $\frac{7}{8}$ of a barrel of flour, worth \$7 a barrel.
22. Find $\frac{5}{9}$ of 3 bushels.
23. Find $\frac{4}{7}$ of 5 oranges.
24. Multiply 3 by $2\frac{1}{2}$.

SOLUTION.— 2 times 3 is 6; $\frac{1}{2}$ of 3 is $1\frac{1}{2}$; $6+1\frac{1}{2}=7\frac{1}{2}$.

25. Multiply 5 by $2\frac{2}{3}$.
26. Find cost of 6 yards of cloth, at $3\frac{1}{2}$ dollars a yard.
27. Find cost of 12 pounds sugar, at $8\frac{1}{3}$ cents per pound.
28. Find 9 times $7\frac{5}{6}$.

WRITTEN.

29. Multiply 24 by $5\frac{3}{8}$.
30. Multiply 32 by $9\frac{3}{4}$.
31. Multiply 108 by $6\frac{5}{9}$.
32. Multiply 144 by $8\frac{7}{12}$.
33. Multiply 84 by $\frac{3}{5}$.
34. Multiply 158 by $\frac{5}{7}$.
35. Multiply 49 by $3\frac{5}{6}$.
36. Multiply 310 by $8\frac{7}{9}$.
37. Multiply 270 by $10\frac{2}{3}$.
38. Multiply 200 by $6\frac{2}{9}$.
39. Multiply 312 by $5\frac{5}{6}$.
40. Multiply 840 by $6\frac{6}{7}$.

41. Find value of 384 acres of land at $\$10\frac{3}{8}$ per acre.

42. How many miles can a ship sail in $18\frac{1}{2}$ days, sailing 179 miles in 1 day?

43. Cost of $40\frac{2}{3}$ tons of hay, at $\$9$ per ton?

CASE III.—To multiply one fraction by another.

ORAL.

143. *1.* What is $\frac{1}{2}$ of $\frac{2}{3}$ of an apple?

ANALYSIS.—As one half of 2 *units* is 1 *unit*, so one half of 2 *thirds* is 1 *third*.

2. What is $\frac{1}{2}$ of $\frac{1}{2}$ of an orange?

Evidently *one fourth* of an orange; since one half equals two fourths, one half of one half is one half of two fourths, which is one fourth.

3. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

ANALYSIS.—This is equivalent to: Find $\frac{4}{5}$ of $\frac{2}{3}$. $\frac{1}{3}$ of $\frac{1}{3}$ is $\frac{1}{15}$; $\frac{4}{5}$ of $\frac{1}{3}$ are 4 times $\frac{1}{15}$, or $\frac{4}{15}$; $\frac{4}{5}$ of $\frac{2}{3}$ are twice as much as $\frac{4}{5}$ of $\frac{1}{3}$, that is, are $\frac{8}{15}$.

SOLUTION.— $\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{3\times5}=\frac{8}{15}$.

Rule.—*Divide the product of the numerators by the product of the denominators.*

4. Multiply $\frac{3}{5}\times\frac{1}{3}$.
5. Multiply $\frac{5}{8}\times\frac{1}{8}$.
6. Multiply $\frac{3}{7}\times\frac{2}{5}$.
7. Multiply $\frac{4}{5}\times\frac{3}{8}$.
8. Multiply $\frac{5}{6}\times\frac{7}{8}$.
9. Multiply $\frac{2}{5}\times\frac{5}{7}$.

10. What part of an apple is $\frac{3}{4}$ of $\frac{1}{2}$ of it?

11. What part of a journey is $\frac{2}{3}$ of $\frac{5}{7}$ of it?

12. Find cost of $\frac{3}{8}$ of a yard of silk, at $\frac{3}{4}$ of a dollar a yard.

13. A man owning $\frac{3}{4}$ of a ship, sold $\frac{1}{5}$ of his share. What part of the ship did he sell?

14. If a man earn $\frac{1}{8}$ of a dollar in 1 hour, how much can he earn in $\frac{3}{4}$ of an hour?

15. Multiply $\frac{2}{3}\times\frac{3}{4}$.

SUGGESTIONS.—Cancel factors common to both numerator and denominator. The product of the numerators is a *dividend;* the product of the denominators is a *divisor.* The rejection of factors common to both dividend and divisor does not affect the quotient. (Art. 105).

16. Multiply $\frac{3}{4}\times\frac{4}{6}$.
17. Multiply $\frac{5}{9}\times\frac{9}{8}$.
18. Multiply $\frac{1}{2}\times\frac{4}{7}$.
19. Multiply $\frac{3}{8}\times\frac{4}{9}$.
20. Multiply $\frac{4}{5}\times\frac{15}{16}$.
21. Multiply $\frac{4}{3}\times\frac{3}{4}$.
22. Multiply $\frac{5}{12}\times\frac{3}{10}$.
23. Multiply $\frac{3}{10}\times\frac{4}{9}$.
24. Multiply $\frac{4}{7}\times\frac{8}{8}$.
25. Multiply $\frac{7}{8}\times\frac{2}{5}$.

26. Multiply $2\frac{1}{2}$ by $\frac{5}{8}$.

SUGGESTION.—Reduce $2\frac{1}{2}$ to an improper fraction.

27. Multiply $3\frac{1}{3}$ by $1\frac{1}{2}$.

SUGGESTION.—Reduce both factors to improper fractions.

28. Multiply $1\frac{2}{5}$ by $3\frac{1}{2}$.
29. Multiply $2\frac{1}{3}$ by $1\frac{3}{4}$.

WRITTEN.

30. Multiply $11\frac{1}{2}$ by $\frac{7}{8}$.
31. Multiply $\frac{9}{11}$ by $\frac{5}{12}$
32. Multiply $4\frac{1}{5}\times\frac{6}{7}$.
33. Multiply $11\frac{3}{7}\times3\frac{2}{5}$.
34. Multiply $4\frac{5}{12}$ by $5\frac{4}{5}$.
35. Multiply $3\frac{6}{7}$ by $8\frac{3}{10}$.
36. Multiply $8\frac{2}{3}$ by $6\frac{2}{5}$.
37. Multiply $9\frac{1}{8}$ by $2\frac{2}{5}$.

38. Cost of $80\frac{3}{4}$ acres of land at $\$2\frac{1}{2}$ per acre?
39. Cost of $112\frac{1}{2}$ pounds sugar at $8\frac{1}{3}$ cents per pound?
40. Cost of $95\frac{3}{5}$ yards cloth at $\$3\frac{2}{3}$ per yard?
41. $\frac{3}{8}\times\frac{2}{3}\times\frac{7}{12}\times\frac{4}{5}\times\frac{3}{7}=?$

Use Cancellation.

42. What part of an acre of land is $\frac{3}{4}$ of $\frac{7}{9}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of it?

NOTE.—The word *of* is here equivalent to the sign ×. Fractions such as the above are called *compound fractions,* to distinguish them from such fractions as $\frac{3}{4}$, $\frac{7}{9}$, etc., which are called *simple* fractions.

144. A ***Simple Fraction*** is a fraction whose terms are both integers.

145. A ***Compound Fraction*** is a fraction of a fraction.

43. Find $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{14}{18}$ of $\frac{3}{5}$.

44. Find $\frac{3}{4}$ of $\frac{2}{11}$ of $\frac{5}{3}$ of $\frac{22}{30}$.

45. Find $\frac{7}{9}$ of $\frac{8}{13}$ of $\frac{3}{8}$ of $\frac{5}{21}$ of $\frac{6}{7}$.

46. Simplify $\frac{3}{4}\times\frac{2}{9}\times\frac{4}{7}\times\frac{2}{11}\times\frac{5}{8}$.

47. Simplify $\frac{2}{5}\times 3\frac{1}{2}\times 1\frac{4}{9}\times\frac{2}{7}\times\frac{4}{13}$.

DIVISION OF FRACTIONS.

CASE I.—To divide a fraction by an integer.

ORAL.

146. *1.* Divide $\frac{5}{8}$ by 5.

This is equivalent to: Find $\frac{1}{5}$ of $\frac{5}{8}$.

SOLUTION.—One fifth of 5 eighths is 1 eighth.

2. Divide $\frac{4}{9}$ by 4.
3. Divide $\frac{6}{7}$ by 3.
4. Divide $\frac{12}{11}$ by 4.
5. Divide $\frac{9}{10}$ by 3.
6. Divide $\frac{10}{11}$ by 5.
7. Divide $\frac{18}{5}$ by 6.

8. Divide $\frac{5}{8}$ by 2.

ANALYSIS.—Following the method used in example 1, that is, dividing the numerator of the fraction by the integer, $\frac{5}{8}\div 2=\frac{5\div 2}{8}=\frac{2\frac{1}{2}}{8}$. This result is correct, but is not in the form of a simple fraction. Instead, therefore, of dividing the numerator by 2, thus taking one half as many fractional units, it is better to take the *same* number of fractional units, each half as large. That is, multiply the denominator by 2: $\frac{5}{8}\div 2=\frac{5}{8\times 2}=\frac{5}{16}$. It is also evident, that since $\frac{1}{16}$ is $\frac{1}{2}$ of $\frac{1}{8}$, $\frac{5}{16}$ is $\frac{1}{2}$ of $\frac{5}{8}$; that is, $\frac{5}{8}\div 2=\frac{5}{16}$.

SOLUTION.— $\frac{5}{8}\div 2=\frac{5}{8\times 2}=\frac{5}{16}$.

Rule.—*Divide the numerator*, OR *multiply the denominator, by the integer.*

NOTE.—Observe, that as division is the opposite of multiplication, the rule for dividing a fraction by an integer is the opposite of the rule for multiplying a fraction by an integer.

9. Divide $\frac{1}{2}$ by 2.
10. Divide $\frac{2}{3}$ by 3.
11. Divide $\frac{5}{6}$ by 2.
12. Divide $\frac{3}{5}$ by 4.
13. Divide $\frac{5}{9}$ by 3.
14. Divide $\frac{7}{8}$ by 5.
15. Divide $\frac{3}{7}$ by 4.
16. Divide $\frac{4}{5}$ by 5.

17. If $\frac{2}{3}$ of a melon be divided equally between 2 boys, what part of the melon will each boy have?

18. If $\frac{3}{8}$ of a melon be divided equally between 2 boys, what part of the melon will each boy have?

19. If $\frac{5}{9}$ of a gallon of wine be divided equally among 6 men, how much will each man have?

20. Divide $12\frac{3}{4}$ by 3.

SOLUTION.— $12 \div 3 = 4$; $\frac{3}{4} \div 3 = \frac{1}{4}$; $4 + \frac{1}{4} = 4\frac{1}{4}$.

21. Divide $12\frac{3}{4}$ by 5.

SOLUTION I.—Reduce the mixed number, $12\frac{3}{4}$ to an improper fraction; $12\frac{3}{4} = \frac{51}{4}$. $\frac{51}{4} \div 5 = \frac{51}{4 \times 5} = \frac{51}{20} = 2\frac{11}{20}$.

SOLUTION II.—Take from the dividend $12\frac{3}{4}$, the greatest multiple of the divisor, 5, contained in it; $12\frac{3}{4} = 10 + 2\frac{3}{4}$. Divide each part of the dividend thus separated, and add the quotients; $10 \div 5 = 2$; $2\frac{3}{4} \div 5 = \frac{11}{4} \div 5 = \frac{11}{4 \times 5} = \frac{11}{20}$. $2 + \frac{11}{20} = 2\frac{11}{20}$.

22. Divide $3\frac{2}{3}$ by 2.
23. Divide $5\frac{1}{2}$ by 3.
24. Divide $10\frac{2}{5}$ by 4.
25. Divide $15\frac{5}{6}$ by 5.
26. Divide $12\frac{3}{8}$ by 3.
27. Divide $14\frac{3}{4}$ by 6.
28. Divide $15\frac{4}{5}$ by 7.
29. Divide $20\frac{2}{3}$ by 8.

30. $8\frac{3}{4}$ apples were divided equally among 3 girls. How many did each receive?

31. $12\frac{4}{5}$ bushels of oats were divided equally among 5 horses. How much did each horse receive?

32. If $9\frac{4}{5}$ barrels of apples were divided equally among 8 families, how many would each receive?

33. Divide $10\frac{6}{7}$ sacks of flour equally among 6 persons.

WRITTEN.

34. Divide $\frac{144}{18}$ by 12.
35. Divide $\frac{108}{20}$ by 9.
36. Divide $\frac{85}{7}$ by 10.
37. Divide $\frac{39}{15}$ by 12.
38. Divide $\frac{95}{15}$ by 8.
39. Divide $125\frac{3}{4}$ by 6.
40. Divide $240\frac{5}{9}$ by 8.
41. Divide $106\frac{2}{7}$ by 12.
42. Divide $209\frac{5}{8}$ by 10.
43. Divide $312\frac{7}{12}$ by 11.

44. If a man walk $39\frac{2}{5}$ miles in 9 hours, how many miles can he walk in 1 hour?

45. If a locomotive can run $496\frac{7}{12}$ miles in 12 hours, how many miles can it run in 1 hour?

46. If 11 acres of land cost $\$125\frac{3}{4}$, what is the cost of one acre?

47. If 9 oxen cost $\$320\frac{2}{5}$, what is the cost of one ox?

CASE II.—To divide an integer by a fraction.

ORAL.

147. *1.* How many halves are there in 1? How many thirds? Fourths?

2. How many fifths are there in 2? In 4? In 5?

3. How may times is $\frac{1}{5}$ contained in 1 unit?

SOLUTION.—There are 5 fifths in 1 unit. $\frac{5}{5} \div \frac{1}{5} = 5 \div 1 = 5$.

4. Divide 4 by $\frac{1}{5}$.

SOLUTION.— $4 = \frac{20}{5}$; $\frac{20}{5} \div \frac{1}{5} = 20 \div 1 = 20$.

5. Divide 4 by $\frac{3}{5}$.

SOLUTION I.— $4 = \frac{20}{5}$; $\frac{20}{5} \div \frac{3}{5} = 20 \div 3 = 6\frac{2}{3}$.

SOLUTION II.—The same result may be obtained by multiplying the integer by the *denominator*, and dividing the product by the numerator, of the fraction; that is, by multiplying the integer by the fraction *inverted*.

Thus, $4 \div \frac{3}{5} = 4 \times \frac{5}{3} = \frac{4 \times 5}{3} = \frac{20}{3} = 6\frac{2}{3}$.

Rule.—*Multiply the integer by the fraction inverted.*

6. Divide 3 by $\frac{1}{4}$.

7. Divide 5 by $\frac{3}{4}$.

8. Divide 4 by $\frac{2}{5}$.

9. Divide 8 by $\frac{3}{4}$.

10. Divide 6 by $\frac{5}{9}$.

11. Divide 7 by $\frac{5}{12}$.

12. Divide 2 oranges among some girls, so that each girl shall have $\frac{2}{3}$ of an orange. How many girls will there be?

13. How many boys can have $\frac{3}{4}$ of an apple each, if 6 apples are divided among them?

14. How many yards of ribbon, at $\frac{2}{5}$ of a dollar per yard, can be bought for 3 dollars?

15. How many hours will it take a boy to walk 8 miles, if he walk $\frac{5}{6}$ of a mile an hour?

16. Divide 4 by $2\frac{1}{2}$.

SOLUTION I.—Reducing each number to halves, $4=\frac{8}{2}$, $2\frac{1}{2}=\frac{5}{2}$; $\frac{8}{2}\div\frac{5}{2}=8\div5=1\frac{3}{5}$.

SOLUTION II.—Reduce $2\frac{1}{2}$ to halves; $2\frac{1}{2}=\frac{5}{2}$. Multiply the integer 4 by the fraction $\frac{5}{2}$ inverted, that is, by $\frac{2}{5}$. $4\times\frac{2}{5}=\frac{4\times2}{5}=\frac{8}{5}=1\frac{3}{5}$.

17. Divide 3 by $1\frac{2}{3}$.
18. Divide 5 by $3\frac{1}{4}$.
19. Divide 8 by $4\frac{2}{7}$.
20. Divide 7 by $3\frac{3}{4}$.

WRITTEN.

21. Divide 8 by $\frac{5}{12}$.
22. Divide 12 by $\frac{4}{9}$.
23. Divide 10 by $\frac{7}{11}$.
24. Divide 9 by $\frac{8}{9}$.
25. Divide 6 by $2\frac{5}{9}$.
26. Divide 12 by $3\frac{4}{7}$.
27. Divide 9 by $12\frac{1}{2}$.
28. Divide 7 by $10\frac{2}{3}$.

29. How many bushels of oats, at $\frac{2}{5}$ of a dollar a bushel, can be bought for \$200?

30. How many yards of cloth, at $\$4\frac{1}{2}$ per yard, can be bought for \$90?

CASE III.—To divide a fraction by a fraction.

ORAL.

148. *1.* How many fourths in $\frac{1}{2}$? How many sixths?

2. How many times is $\frac{1}{4}$ contained in $\frac{1}{2}$?

3. How many times is $\frac{1}{6}$ contained in $\frac{5}{6}$?

4. Divide $\frac{6}{8}$ by $\frac{3}{8}$.

NOTE.—It is evident that as 3 *units* are contained in 6 *units* 2 times, so are 3 *eighths* contained in 6 *eighths* 2 times.

5. Divide $\frac{3}{4}$ by $\frac{2}{5}$.

SOLUTION I.—Reduce the fractions to equivalent fractions having the least common denominator (Art. 138). $\frac{3}{4}=\frac{15}{20}$; $\frac{2}{5}=\frac{8}{20}$. Divide the numerator of the dividend by the numerator of the divisor: $15\div8=1\frac{7}{8}$.

SOLUTION II.—The same result may be obtained by multiplying the dividend by the divisor inverted.

Thus, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = 1\frac{5}{8} = 1\frac{7}{8}$.

Rule I.—*Reduce the fractions to equivalent fractions having a common denominator. Divide the numerator of the dividend by the numerator of the divisor.*

Rule II.—*Multiply the dividend by the divisor inverted.*

6. Divide $\frac{3}{8}$ by $\frac{1}{4}$.
7. Divide $\frac{4}{5}$ by $\frac{2}{3}$.
8. Divide $\frac{4}{7}$ by $\frac{2}{9}$.
9. Divide $\frac{3}{4}$ by $\frac{3}{5}$.
10. Divide $\frac{2}{3}$ by $\frac{4}{5}$.
11. Divide $\frac{5}{6}$ by $\frac{3}{8}$.
12. Divide $\frac{3}{4}$ by $\frac{5}{7}$.
13. Divide $\frac{5}{8}$ by $\frac{2}{5}$.

14. Divide $\frac{5}{9}$ by $1\frac{1}{2}$.

SOLUTION.—Reduce $1\frac{1}{2}$ to an improper fraction; $1\frac{1}{2} = \frac{3}{2}$. $\frac{5}{9} \div \frac{3}{2} = \frac{5}{9} \times \frac{2}{3} = \frac{10}{27}$.

15. Divide $1\frac{2}{3}$ by $3\frac{3}{4}$.

SOLUTION.—Reduce to improper fractions; $1\frac{2}{3} = \frac{5}{3}$; $2\frac{3}{4} = \frac{11}{4}$. $\frac{5}{3} \div \frac{11}{4} = \frac{5}{3} \times \frac{4}{11} = \frac{20}{33}$.

16. Divide $\frac{3}{5}$ by $1\frac{1}{2}$.
17. Divide $\frac{2}{9}$ by $2\frac{1}{5}$.
18. Divide $2\frac{1}{2}$ by $3\frac{1}{4}$.
19. Divide $3\frac{1}{3}$ by $1\frac{2}{5}$.

20. Into how many pieces, each $\frac{3}{4}$ of an inch long, may a stick $2\frac{1}{2}$ inches long be cut?

21. How many times may a cup holding $\frac{2}{3}$ of a pint be filled from a jar holding $5\frac{1}{3}$ pints?

22. How many pencils, costing $\frac{4}{5}$ of a cent each, may be bought for $6\frac{2}{5}$ cents?

23. Into how many balls, each weighing $1\frac{1}{2}$ ounces, may a piece of lead weighing $10\frac{3}{8}$ ounces be divided?

WRITTEN.

24. Divide $7\frac{7}{12}$ by $\frac{5}{8}$.
25. Divide $9\frac{9}{11}$ by $\frac{7}{9}$.
26. Divide $3\frac{2}{3}$ by $6\frac{1}{2}$.
27. Divide $5\frac{4}{7}$ by $8\frac{3}{4}$.
28. Divide $125\frac{5}{9}$ by $2\frac{3}{11}$.
29. Divide $92\frac{5}{6}$ by $12\frac{4}{7}$.
30. Divide $32\frac{1}{3}$ by $64\frac{2}{5}$.
31. Divide $34\frac{2}{9}$ by $81\frac{6}{7}$.

32. How many bottles, each containing $2\frac{1}{4}$ pints, may be filled from a vessel containing $204\frac{2}{3}$ pints?

33. How many penholders, worth $2\frac{5}{6}$ cents each, may be bought for $50\frac{3}{4}$ cents?

34. How many dozen lead pencils, worth $\$1\frac{1}{4}$ per dozen, can be bought for $\$24\frac{2}{3}$?

REVIEW QUESTIONS.

How are fractions having a common denominator added or subtracted? What must be done with fractions which do not have a common denominator before they can be added or subtracted? What two methods of multiplying a fraction by an integer? When is the first method used? The second? State two methods of multiplying an integer by a fraction. When is the first method used? The second? How is one fraction multiplied by another fraction? How is cancellation applied to this process? Why? How is a mixed number multiplied by a fraction or a mixed number? How is a fraction divided by an integer? When is one method used? When the other? How is an integer divided by a fraction? How is a fraction divided by a fraction? How is a mixed number divided by a fraction or mixed number?

MISCELLANEOUS PROBLEMS.

ORAL.

1. Change $3\frac{1}{2}$ to halves.
2. Change $5\frac{3}{8}$ to 8ths.
3. Reduce $7\frac{1}{4}$ to 7ths.
4. Reduce $\frac{36}{4}$ to integers.
5. Reduce $\frac{80}{7}$ to mixed number.
6. Change $\frac{50}{17}$ to whole or mixed number.
7. Reduce $\frac{3}{4}$ to 28ths.
8. Reduce $\frac{6}{30}$ to 10ths.
9. Reduce $\frac{2}{9}$ and $\frac{2}{3}$ to 36ths.
10. Reduce $3\frac{2}{7}$ to an improper fraction.

11. Reduce to lowest terms $\frac{30}{25}$, $\frac{7}{14}$, $\frac{32}{48}$.

12. Reduce to lowest terms $\frac{40}{50}$, $\frac{60}{72}$, $\frac{81}{108}$.

13. Reduce to common denominator $\frac{1}{2}$ and $\frac{2}{5}$.

14. Reduce to least common denominator $\frac{2}{3}$ and $\frac{5}{6}$.

15. Reduce to least common denominator $\frac{5}{8}$, $\frac{3}{4}$ and $\frac{3}{5}$.

16. Add $\frac{2}{4}$ and $\frac{3}{8}$.

17. Add $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{5}{12}$.

18. Find difference of $\frac{3}{4}$ and $\frac{7}{12}$.

19. Multiply $\frac{3}{7}$ by $\frac{5}{9}$.

20. Divide $\frac{8}{8}$ by $\frac{2}{3}$.

21. $3\frac{1}{3}$ is $\frac{1}{6}$ of what number?

SOLUTION.— $3\frac{1}{3}$ is $\frac{1}{6}$ of 6 times $3\frac{1}{3}$, which is 20.

22. $2\frac{1}{2}$ is $\frac{1}{4}$ of what number?

23. $12\frac{1}{2}$ is $\frac{1}{8}$ of what number?

24. $6\frac{1}{4}$ is $\frac{1}{12}$ of what number?

25. 8 is $\frac{2}{5}$ of what number?

SOLUTION.—Since 8 is $\frac{2}{5}$ of some number, $\frac{1}{5}$ of that number is $\frac{1}{2}$ of 8, which is 4; and $\frac{5}{5}$, or the whole number, is 5 times 4, which is 20.

26. 9 is $\frac{3}{4}$ of what number?

27. 12 is $\frac{2}{3}$ of what number?

28. \$18 is $\frac{9}{10}$ of the price of a shawl. What is the price of the shawl?

29. John, who is 15 years old, is $\frac{5}{7}$ the age of Henry. How old is Henry?

30. If $\frac{4}{5}$ of a ton of coal is worth \$8, what is the price of 1 ton?

31. After spending $\frac{1}{3}$ of her money, Mary had 24 cents left. How many cents had she at first?

SOLUTION.—Since she spent $\frac{1}{3}$ of her money, she had $\frac{2}{3}$ of it left. $\frac{2}{3}$ of her money equals 24 cents; $\frac{1}{3}$ of her money is $\frac{1}{2}$ of 24 cents, or 12 cents; $\frac{3}{3}$, or the whole, is 3 times 12 cents, which is 36 cents.

32. James lost $\frac{2}{5}$ of his marbles, and then had 30 remaining. How many had he at first?

33. Mr. D. sold $\frac{3}{8}$ of his farm, and then had 500 acres. How many acres had he before he sold?

34. $\frac{1}{7}$ of a school was dismissed at recess, when there were 42 pupils remaining. How many pupils belonged to the school?

35. If $\frac{3}{8}$ of a yard of cloth is worth $\frac{5}{6}$ of a dollar, what is the value of 1 yard?

SOLUTION.—Since $\frac{3}{8}$ of a yard is worth $\frac{5}{6}$ of a dollar $\frac{1}{8}$ of a yard is worth $\frac{1}{3}$ of $\frac{5}{6}$ of a dollar, which is $\frac{5}{18}$ of a dollar, and $\frac{8}{8}$ of a yard, or 1 yard, is worth 8 times $\frac{5}{18}$ of a dollar, which is $\frac{40}{18}$ of a dollar.

36. If $\frac{2}{3}$ of a sack of flour is worth $\frac{7}{8}$ of a dollar, what is 1 sack of flour worth?

37. If $\frac{4}{5}$ of John's money equals $\frac{3}{4}$ of William's money, what part of William's money is John's?

38. $\frac{3}{4}$ is $\frac{2}{3}$ of what number?

39. $3\frac{1}{2}$ is $\frac{4}{5}$ of what number?

40. $4\frac{2}{3}$ is $\frac{7}{5}$ of what number?

41. $\frac{3}{5}$ is the product of $\frac{1}{2}$ and of what other number?

42. $5\frac{1}{4}$ is the product of 7 and of what other number?

43. The divisor is $\frac{2}{3}$, the quotient is $\frac{3}{8}$. What is the dividend?

44. The multiplicand is $2\frac{1}{2}$, the product is $\frac{1}{2}$. What is the multiplier?

45. The multiplicand is $\frac{7}{9}$, the multiplier is $\frac{2}{5}$. What is the product?

46. $2\frac{2}{3}$ is the product of 3 and of what other number?

47. $7\frac{1}{2}$ is the sum of what two equal numbers?

48. $7\frac{7}{12}$ is the sum of $\frac{1}{8}$ and of what other number?

49. $\frac{4}{5}$ is the difference of $1\frac{1}{4}$ and of what other number?

50. What three equal numbers, when added, will produce $1\frac{7}{8}$?

51. $\frac{1}{2}$ of a certain number exceeds $\frac{1}{5}$ of it by 6. What is the number?

SOLUTION.—The difference of $\frac{1}{2}$ and $\frac{1}{5}$ is $\frac{3}{10}$. 6 is $\frac{3}{10}$ of the

required number. $\frac{1}{10}$ of the number is $\frac{1}{3}$ of 6, which is 2; the number is 10 times 2, or 20.

52. The difference between $\frac{2}{3}$ and $\frac{1}{2}$ of a number is 5. What is the number?

53. \$10 is the difference between $\frac{3}{4}$ and $\frac{1}{3}$ of the cost of a horse. What was the cost of the horse?

54. Mr. E. owns $\frac{3}{4}$ of a ship, and Mr. F. owns $\frac{1}{8}$ of the same ship; Mr. G. owns the remainder. What part of the ship does Mr. G. own?

55. Mr. H. owned $\frac{2}{3}$ of a mill, but sold $\frac{2}{7}$ of his share. What part of the mill did he sell?

56. Mrs. K. had $\frac{5}{6}$ of a gallon of syrup, and gave her neighbor $\frac{2}{7}$ of what she had. What part of a gallon did she give?

57. Mrs. M. had $\frac{5}{6}$ of a gallon of syrup, and gave her neighbor $\frac{2}{7}$ of a gallon. What part of a gallon did she keep?

58. If $\frac{3}{5}$ of a ton of hay cost 6 dollars, what will $\frac{7}{8}$ of a ton cost?

SUGGESTIONS.—Find the cost of $\frac{1}{5}$ of a ton; then of 1 ton; then of $\frac{1}{8}$ of a ton; then of $\frac{7}{8}$ of a ton.

59. If $\frac{5}{6}$ of an acre of land cost 25 dollars, what will $1\frac{1}{2}$ acres cost?

60. How many barrels of flour, at $\$5\frac{1}{2}$ per barrel, can be bought for \$33?

WRITTEN.

61. Reduce $321\frac{5}{9}$ to 9ths.

62. Reduce $\frac{78}{12}$ to integer or mixed number.

63. $7\frac{5}{6}$ is $\frac{1}{12}$ of what number?

64. $20\frac{2}{3}$ is $\frac{2}{7}$ of what number?

65. Find sum of $2\frac{1}{2}$, $5\frac{2}{3}$, and $8\frac{4}{5}$.

66. Find difference of $8\frac{3}{7}$ and $5\frac{6}{11}$.

67. Reduce to lowest terms $\frac{14}{77}$ and $\frac{69}{96}$.

68. What number is $3\frac{1}{2}$ less than $5\frac{5}{9}$?

69. What number is $2\frac{3}{7}$ more than $8\frac{3}{4}$?

70. What is $\frac{8}{8}$ of $\frac{5}{6}$ of $\frac{4}{5}$ of $\frac{1}{2}$?

71. The multiplier is $2\frac{4}{5}$, the multiplicand is $8\frac{2}{3}$. Find the product.

72. The divisor is $3\frac{2}{7}$, the quotient is $\frac{5}{9}$. What is the dividend?

73. Mr. A. had a farm of 640 acres. He sold $\frac{5}{8}$ of it for $25 per acre. What did he receive?

74. Find cost of 200 barrels of flour at $6\frac{1}{4}$ per barrel.

75. A man bought $\frac{1}{5}$ of $\frac{3}{4}$ of 400 acres of land. How many acres did he buy?

76. A man lost $23\frac{3}{4}$, which was $\frac{2}{3}$ of his money. How much had he at first?

77. How many bushels of wheat, worth $1\frac{1}{8}$ per bushel, may be exchanged for 175 bushels corn, worth $\frac{3}{8}$ of a dollar per bushel?

78. If $\frac{2}{3}$ of a yard of cloth cost $4\frac{2}{5}$ dollars, what will $3\frac{1}{2}$ yards cost?

79. At $\frac{3}{8}$ of a dollar a pound, how many pounds of coffee can be bought for $3\frac{5}{6}$?

80. Find cost of $3\frac{3}{4}$ pounds of tea, at $1\frac{1}{5}$ per pound.

81. Mr. A. bought 5 houses; the first cost $3475.75; the second $2107.50; the third $4500; the fourth $1787.875; the fifth $2000. Find the average cost of each.

82. $\frac{2}{3}$ of 8100 is what part of 3375?

83. Mr. D. lost $400, and had $\frac{5}{9}$ of his money remaining. How much did he have at first?

84. If $\frac{3}{4}$ of a vessel is worth $27000, what is the value of $\frac{3}{4}$ of the vessel?

85. A number increased by $\frac{2}{5}$ of itself is 28; what is the number?

86. The product of 3 numbers is $29\frac{2}{3}$; one of the numbers is $5\frac{4}{5}$; another is $9\frac{1}{2}$. What is the third number?

87. $\frac{5}{8}$ of a farm is 48 acres less than $\frac{2}{3}$ of it. How many acres are in the farm?

88. Mr. C. has $\frac{2}{5}$ of his property in city real estate, $\frac{1}{8}$ of it in farming lands, and the remainder, $5700, in bank stock. What is he worth?

SECTION X.

DECIMAL FRACTIONS.

Art. 149. When a unit of any kind is divided into 10 equal parts, each part is called *one tenth*. When *one tenth* is divided into ten equal parts, each part is *one tenth of one tenth*, and is called *one hundredth*.

When *one hundredth* is divided into ten equal parts, each part is *one tenth of one hundredth*, and is called *one thousandth*.

These fractions, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., increase and decrease in the ratio of 10; that is, each is *one tenth* of the next higher in the scale, and *ten times* the next lower. They are therefore called *decimal fractions*, or, more briefly, *decimals*.

Observe that they increase and decrease exactly as integers increase and decrease. Integers are *decimals*, for the same reason that decimal fractions are decimals; they increase and decrease in the ratio of 10.

150. The denominator of a decimal fraction is always 1, with ciphers annexed; that is, it is always a power of 10. (Art. 108). It is never 20, nor 50, nor 180; but it is always 10, 100, 1000, 10000, etc.

Thus, $\frac{5}{10}$ is a decimal, but $\frac{1}{2}$ and $\frac{10}{20}$, equivalents of $\frac{5}{10}$, are not decimals, because their denominators, 2 and 20, are not powers of 10.

It is not necessary to write the *denominators* of decimal fractions. $\frac{3}{10}$ may be written .3; the period, here called the *decimal point*, distinguishes 3 *tenths* from 3 units. $.3=\frac{3}{10}$; $3.=3$ *units*.

$\frac{3}{100}$ may be written .03; $\frac{3}{1000}$ may be written .003;
$\frac{25}{100}$ may be written .25; $\frac{125}{1000}$ may be written .125.

Observe that when the denominator of a decimal fraction is not written, *two* things are necessary: 1st. The decimal point should be prefixed to the numerator of the fraction; 2nd. The decimal must have as many figures as there are 0's in the written denominator. If there are not as many figures in the numerator as there are 0's in the denominator, 0's must be *prefixed* to the numerator, when the denominator is not written.

Thus, $\frac{5}{10}$=.5; $\frac{18}{100}$=.18; $\frac{4}{1000}$=.004; etc.

151. The *decimal point* is always written at the left of the decimal fraction. It is sometimes called the *separatrix*, because it separates decimal fractions from integers.

Thus, .25=$\frac{25}{100}$; but 2.5=$2\frac{5}{10}$. The number on the left of the decimal point is an integer.

152. A ***Decimal Fraction*** is a fraction whose denominator is a power of 10.

TABLE.

Millions.	Hundred-Thousands.	Ten-Thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.
7	6	5	4	3	2	1	.	1	2	3	4	5	6
INTEGERS.								DECIMAL FRACTIONS.					

READING DECIMALS.

1. Read .32.

SOLUTION I.—The 3 occupies the place of *tenths;* the 2 the place of *hundredths.* 3 *tenths* and 2 *hundredths* are 32 *hundredths.*

SOLUTION II.—Since the denominator of a decimal fraction is always 1 with as many 0's as there are decimal figures, .32 must equal $\frac{32}{100}$, or 32 *hundredths.*

Rule.—*For the numerator, read the decimal as though it were an integer. For the denominator, read the denomination of the right hand figure.*

Read the following:

2. .8; .08; .008; .0008.
3. .35; .48; .408; .567.
4. .896; .4564; .39275.
5. 3.4; 3.43; 89.6756.
6. .0908; 35.0006; 4.50004.

WRITING DECIMALS.

153. *7.* Write 8 thousandths.

SOLUTION.—Since *thousandths* is the name of the third decimal order, three orders of decimals are necessary in writing thousandths. Write the 8, prefixing two 0's and the decimal point, thus, .008.

8. Write 308 millionths.

SOLUTION.—Since *millionths* is the name of the sixth decimal order, six orders of decimals are necessary in writing millionths. Write the 308 as an integer, and prefix three 0's and the decimal point, thus, .000308.

Rule.—*Write the numerator as an integer. Prefix* 0*'s to the numerator, if necessary, until the number of figures in the decimal equals the number of* 0*'s in the denominator. Write the decimal point at the left of the decimal.*

9. Write 29 ten-thousandths.
10. Write 496 thousandths.
11. Write fifty-six hundredths.
12. Write eighty-four millionths.
13. Write 166 ten-thousandths.
14. Write 48 integers, and 64 hundredths.
15. Write 896, and 35 thousandths.
16. Write 9846, and 29 millionths.
17. Write thirty-four, and three hundred eight ten-thousandths.

18. Write 46 tenths.

SOLUTION.—46 tenths$=\frac{46}{10}=4\frac{6}{10}=4.6$.

19. Write 124 hundredths.

20. Write 4986 thousandths.

154. By *annexing* 0 to a decimal, both terms of the fraction are really multiplied by 10, and the value of the decimal is not changed.

Thus, .5=.50; that is, $\frac{5}{10}=\frac{50}{100}$; etc.

PRINCIPLE.

1. Annexing 0's to a decimal does not alter its value.

155. By *prefixing* 0 to a decimal, the denominator is multiplied by 10. The value of the fraction is therefore *divided* by 10.

Thus, $.5=\frac{5}{10}$; but $.05=\frac{5}{100}$; and $\frac{5}{100}$ is *one tenth* of $\frac{5}{10}$.

PRINCIPLE.

2. Prefixing 0's to a decimal divides the decimal by 10 as many times as there are 0's prefixed.

21. Change .4 to hundredths.

22. Change .56 to millionths.

23. Divide .8 by 10.

24. Divide .34 by 100.

25. Divide .49 by 1000.

26. Divide .456 by 100.

REDUCTION OF DECIMALS.

CASE I.—To reduce a decimal fraction to a common fraction.

156. *1.* Reduce .8 to a common fraction.

SOLUTION.— $.8=\frac{8}{10}=\frac{4}{5}$.

2. Reduce .08 to a common fraction.

SOLUTION.— $.08=\frac{8}{100}=\frac{2}{25}$.

Rule.—*Erase the decimal point. Write the denominator, and reduce to lowest terms.*

NOTE.—When the decimal point is erased, the 0's at the *left* of a decimal become valueless, and should be omitted. Thus, 08=8. But .08 does *not* equal 8.

Reduce to common fractions:

3. .4.	*8.* .96.	*13.* .800.
4. .6.	*9.* .75.	*14.* .125.
5. .12.	*10.* .16.	*15.* .375.
6. .25.	*11.* .35.	*16.* .875.
7. .48.	*12.* .40.	*17.* .625.

CASE II.–To reduce a common fraction to a decimal fraction.

157. *1.* Reduce $\frac{3}{4}$ to a decimal.

SOLUTION.— $\frac{3}{4}=\frac{1}{4}$ of 3. 3=30 tenths. $\frac{1}{4}$ of 30 tenths is 7 *tenths*, with a remainder of 2 tenths. 2 tenths=20 hundredths. $\frac{1}{4}$ of 20 hundredths is *5 hundredths*. Therefore, $\frac{1}{4}$ of 3, or $\frac{3}{4}$,=7 *tenths*+5 *hundredths*, or 75 hundredths, or .75.

2. Reduce $\frac{1}{16}$ to a decimal.

SOLUTION.— 1=10 tenths. $\frac{1}{16}$ is $\frac{1}{16}$ of 10 tenths, which is *0 tenths*, with a remainder of 10 tenths; 10 tenths=100 hundredths; $\frac{1}{16}$ of 100 hundredths is *6 hundredths*, with a remainder of 4 hundredths; 4 hundredths=40 thousandths; $\frac{1}{16}$ of 40 thousandths is *2 thousandths*, with a remainder of 8 thousandths; 8 thousandths=80 ten-thousandths; $\frac{1}{16}$ of 80 ten-thousandths is *5 ten-thousandths*. *0 tenths*+*6 hundredths* +*2 thousandths*+*5 ten-thousandths*=.0625. That is, $\frac{1}{16}$=.0625.

Rule.—*Annex* 0*'s to the numerator. Divide this result by the denominator. Place the decimal point at the left of the quotient. If necessary, prefix ciphers to the quotient, in order that there may be as many decimal orders in the quotient as there have been* 0*'s annexed to the numerator.*

Reduce to decimal fractions:

3. $\frac{1}{4}$.	*7.* $\frac{1}{40}$.	*11.* $\frac{7}{20}$
4. $\frac{5}{8}$.	*8.* $\frac{4}{5}$.	*12.* $\frac{7}{8}$.
5. $\frac{3}{5}$.	*9.* $\frac{17}{50}$.	*13.* $\frac{6}{40}$.
6. $\frac{9}{25}$.	*10.* $\frac{11}{16}$.	*14.* $\frac{8}{25}$.

15. Reduce $\frac{1}{3}$ to a decimal.

SOLUTION.—Annex 0's and divide by the denominator, according to the rule. Proceed thus as far as may be desirable, and annex the sign +. Thus, $\frac{1}{3}$=.333+.

Reduce to decimals of 4 figures each:

16. $\frac{2}{3}$. *18.* $\frac{1}{7}$.

17. $\frac{1}{6}$. *19.* $\frac{1}{9}$.

REVIEW QUESTIONS.

What is a decimal fraction? Are the denominators of decimal fractions usually written? Why is it not necessary to write them? What is the decimal point? Why is it sometimes called the separatrix? Repeat the rule for reading decimals. Rule for writing decimals. What are the two principles of decimals given? How may a decimal fraction be reduced to a common fraction? How may a common fraction be reduced to a decimal fraction?

ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS.

ORAL.

158. *1.* Add .6 and .9.

SOLUTION.—The sum of 6 tenths and 9 tenths is 15 tenths, equal to 1 and 5 tenths, or 1.5.

2. Add .7 and .4.

3. Add .21 and .12.

4. Add .3 and .05.

SOLUTION.— .3=.30; .30+.05=.35.

5. Add .5 and .09.

6. Add .12 and .014.

159. The principles of addition and subtraction of decimal fractions and of integers are the same, because both are *decimal;* that is, 10 units of any order are equal to 1

unit of the next higher order. When numbers are to be added or subtracted, figures of the same order should be written in the same column; that is, units should be written under units, tenths under tenths, hundredths under hundredths, etc. All the decimal points will, of course, be in column.

WRITTEN.

7. Add 8.5, 25.8395, 69., 4.37, and .075.

PROCESS.

8.5
25.8395
69.
4.37
.075

107.7845

SOLUTION.—Write the numbers so that the decimal points shall all be in the same column. This will place figures of the same order in column.

Add as integers, writing the decimal point between the integer and the decimal fraction in the result.

8. From 237.5 subtract 68.625.

PROCESS.

237.500
68.625

168.875

SOLUTION.—Write the numbers so that figures of the same order shall stand in the same column. Subtract as in integers, placing the decimal point on the left of the tenths in the remainder.

NOTE.—In the process, the .5 have been reduced to .500 by annexing two 0's. It is not necessary, however, to write the 0's.

Rule.— *Write the numbers so that figures of the same order shall stand in the same column. Add or subtract as in integers. Place the decimal point at the left of the tenths in the result.*

9. Add 83.4, 9.64, 324.5, and .8946.

10. Add .375, .0049, 6.2, 54., and 3.03.

11. Subtract 23.45 from 198.62.

12. Subtract 5.946 from 594.6.

13. From the sum of 845, 9.64, and .04, subtract the sum of 4.3 and 50.656.

160. In United States money, the *dollar* is the *unit;*

cents are *hundredths*, and occupy the two places on the *right* of the decimal point; *mills* are *thousandths*, and occupy the third decimal place. The sign of U. S. money is $.

Thus—One dollar is written - - - $1.00 or $1.
One cent is written - - - - .01 or 1¢.
Ten cents are written - - - .10 or 10¢
One mill is written - - - - .001
Ten dollars, thirty-seven cents and five mills are written - - - - $10.375 or $10 37½.

(For fractional parts of a dollar, see page 95.)

FRACTIONAL PARTS OF A CENT.

½ cent =$.005. ⅓ cent =$.003⅓.
¼ cent = .0025. ⅙ cent = .001⅔.

14. Add $25.50, $8.75, $492.00, and $3.25.

15. Add $50., $125.875, $.625, and $35.00.

16. Add $310., $3.10, $49., $.10, and $.007

17. Add $7½, $3¼, $10⅕, and $¾.

SUGGESTION.—Change the fractions to decimals, in accordance with the table, page 95. Thus, $7½=$7.50; $3¼=$3.25; etc.

18. Add $247.50, $29.75, $8⅘, and $9⅝.

19. Find difference of $100. and $75½.

20. Add $1.37½, $2.25, $.12½ and $3½.

SUGGESTION.—Change the fractions of a cent to mills.

21. Add $.50, $.80, $.06¼, $.37½, and $.1675.

22. From $25½ subtract $15.62½.

23. A horse was bought for $150. and sold for $170.50. What was the gain?

24. A man bought a load of hay for $18.75, another for $13.25. He sold the two loads for $35. Did he gain or lose, and how much?

25. A house was bought for $3420.75. For how much must it be sold to gain $175½?

26. John spent 85 cents for a knife, one dollar and a half for a sled, and a quarter of a dollar for a ball. What change should he have received from a $5 bill?

MULTIPLICATION OF DECIMAL FRACTIONS.

ORAL.

161. *1.* 5 times .1 are how many tenths?

2. 3 times .3 are how many tenths?

3. 4 times .6 are how many tenths?

SOLUTION.—4 times .6 are 24 tenths, equal to 2 units and 4 tenths, or 2.4.

4. Multiply .8 by 6; by 7; by 8.

5. Multiply .09 by 5; by 6; by 7.

NOTE.—In decimals, as in integers, the product is the same denomination, or kind, as the multiplicand.

6. Multiply 1.2 by 3; by 4; by 5.

7. Multiply .1 by .1.

SOLUTION.— $.1 \times 1 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$, that is, .01.

8. Multiply .3 by .03.

SOLUTION.— $.3 \times .03 = \frac{3}{10} \times \frac{3}{100} = \frac{9}{1000}$, or .009.

NOTE I. — In the multiplication of one decimal fraction by another decimal fraction, there is really the same process as in the multiplication of any fraction by another fraction; that is, the numerator of the product is the product of the numerators of the factors; and the denominator of the product is the product of the denominators of the factors. Since the denominator of every decimal fraction is a power of 10, the product of the denominators is that power of 10 which contains as many 0's as both, or all, of the denominators of the factors. Thus, $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$; $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$; $\frac{1}{100} \times \frac{1}{100} = \frac{1}{10000}$; etc. In multiplying one decimal fraction by another, therefore, it is necessary to find the product of the *numerators* only, and make the number of decimal orders in the product of the numerators equal to the number in *both* of the factors. Thus, $.3 \times .5 = .15$.

NOTE II. — It is frequently necessary to *prefix* 0's to the product of the numerators, in order to secure the requisite number of decimal orders. Thus, $.1 \times .1 = .01$; $.01 \times .01 = .0001$; etc.

PRINCIPLE.

3. The number of decimal orders in the product of two or more decimal factors is always equal to the number of decimal orders in all the factors.

WRITTEN.

9. Multiply .325 by 5.

PROCESS.

.325
5
1.625

SOLUTION.— 5 times .325=5 times $\frac{325}{1000}=\frac{1625}{1000}=$ 1.625.

10. Multiply .325 by .05.

PROCESS.

.325
.05
.01625

SOLUTION.— $.325\times.05=\frac{325}{1000}\times\frac{5}{100}=\frac{1625}{100000}=$ .01625.

Rule.—*Multiply as in integers, and make the number of decimal orders in the product equal to the number of decimal orders in both factors. If necessary, prefix* 0*'s.*

Multiply:

11. .47 by 8.	*16.* .0324 by .25.
12. 1.35 by 24.	*17.* 9873 by .041.
13. 38.96 by 1.56.	*18.* 50.94 by $33\frac{1}{3}$.
14. .1324 by .15.	*19.* $327.50 by 24.
15. 2.549 by .032.	*20.* $84.675 by 12.

21. Find cost of 25 yards of cloth, worth $4.25 per yard.

22. Find value of 3625 bushels of wheat, worth $1.12\frac{1}{2}$ per bushel.

23. Find cost of 2450 acres of land, worth $28.50 per acre.

24. Multiply 1.8456 by 100.

SUGGESTION.—Simply remove the decimal point two places to the right.

25. Multiply 93.426 by 1000.

26. Find product of .18433 by 100.

27. Multiply .4 by 100.

SUGGESTION.—Removing the decimal point one place to the right

multiplies by 10; erasing this point and annexing 0 multiplies again by 10.

28. Multiply 3.5 by 100.

29. Multiply 54.69 by 1000.

DIVISION OF DECIMAL FRACTIONS.

ORAL.

162. *1.* Divide .6 by 2.

SOLUTION.—Dividing by 2 is finding one half. One half of .6 is .3.

2. Divide .8 by 2; by 4.

3. Divide .12 by 2; by 3; by 4.

4. Divide 3.6 by 9.

SOLUTION.— 3.6=36 tenths. $\frac{1}{9}$ of 36 tenths is 4 tenths, or .4.

5. Divide 5.6 by 8; by 7.

6. Divide 2.4 by 3; by 4; by 6; by 8.

7. Divide 4.8 by 4; by 6; by 8; by 12.

163. The difficulty in division of decimals is the placing of the decimal point in the quotient. But it should be borne in mind that division is the converse of multiplication; that the divisor and quotient are factors of the dividend. Hence, the number of decimal orders in the divisor and quotient must be equal to the number of decimal orders in the dividend. Therefore, the decimal orders in the quotient must equal the excess of the decimal orders in the dividend over those in the divisor.

That is, if there are 5 decimal places in the dividend, and 2 decimal places in the divisor, there must be 3 decimal places in the quotient.

In each of the three preceding examples, there is *one* decimal place in the dividend, *none* in the divisor; therefore, there must be *one* decimal place in the quotient

NOTE.—The correctness of the pointing in the quotient may always be tested by *proving* the work; since the dividend must be the product of divisor and quotient.

PRINCIPLE.

4. The decimal orders in the divisor and quotient are equal in number to the decimal orders in the dividend.

164. The division of a decimal fraction or mixed number by a decimal fraction may be considered in three cases.

1. When the decimal orders in the dividend and divisor are *equal* in number.

2. When the decimal orders in the dividend are *less* in number than those in the divisor.

3. When the decimal orders in the dividend *exceed* in number those in the divisor.

CASE I.

165. *1.* Divide .9 by .3.

SOLUTION.— $.9 \div .3 = \frac{9}{10} \div \frac{3}{10} = 9 \div 3 = 3$.

2. Divide .7 by .3.

SOLUTION.— $.7 \div .3 = \frac{7}{10} \div \frac{3}{10} = 7 \div 3 = 2\frac{1}{3}$. The common fraction $\frac{1}{3}$ may be changed to a decimal (Art. 157).

NOTE.—When dividend and divisor are fractions having a common denominator, their quotient is the quotient of the numerator of the dividend divided by the numerator of the divisor (Art. 148). *When dividend and divisor are decimal fractions having the same number of decimal orders, they have a common denominator; and their quotient is the quotient of their numerators, and is an integer or a mixed number.* When the dividend is a multiple of the divisor, the quotient is an integer. Thus, $2.4 \div .8 = 3$. When the dividend is *not* a multiple of the divisor, the quotient is a mixed number. Thus, $2.5 \div .8 = 3\frac{1}{8}$, or 3.125.

3. Divide .8 by .4.
4. Divide .6 by .2.
5. Divide .18 by .06.
6. Divide .24 by .08.
7. Divide 3.6 by .9.
8. Divide 7.2 by .8.
9. Divide 1.44 by .12.
10. Divide 1.21 by .11.

11. Divide .21 by .04.

SOLUTION.— $.21 \div .04 = \frac{21}{100} \div \frac{4}{100} = 21 \div 4 = 5\frac{1}{4}$ or 5.25.

12. Divide .45 by .06; by .07; by .08; by .09.

CASE II.

166. *1.* Divide .8 by .04.

SOLUTION.—Annex a sufficient number of 0's to the dividend to make the decimal places in the dividend equal to those in the divisor. Then proceed as in Case I. .8=.80. $.80 \div .04 = \frac{80}{100} \div \frac{4}{100} = 80 \div 4 = 20$.

2. Divide .9 by .004.

SOLUTION.—$.9 = .900$; $.900 \div .004 = \frac{900}{1000} \div \frac{4}{1000} = 900 \div 4 = 225$.

Divide:

3. .9 by .03.	*6.* .2 by .04.	*9.* .5 by .002.
4. .6 by .02.	*7.* .4 by .08.	*10.* .7 by .004.
5. .1 by .05.	*8.* .3 by .06.	*11.* .12 by .006.

CASE III.

167. *1.* Divide .24 by .3.

SOLUTION.— $24 \div .3 = \frac{24}{100} \div \frac{3}{10} = \frac{24}{100} \times \frac{10}{3} = \frac{8}{10} = .8$.

NOTE.—The denominator of the quotient is the quotient of 100÷10=10; that is, the denominator of the quotient contains as many 0's as the 0's in the denominator of the dividend exceed the 0's in the denominator of the divisor. Or, when the fractions are written decimally, *the quotient contains as many decimal places as the decimal places in the dividend exceed those in the divisor.*

Divide:

2. .35 by .7.	*6.* .144 by .12.	*10.* .0044 by .11.
3. .42 by .6.	*7.* .80 by .8.	*11.* .096 by .12.
4. .81 by .9.	*8.* .056 by .7.	*12.* .0024 by .003.
5. .63 by .7.	*9.* .0072 by .08.	*13.* .0049 by .007.

DIVISION OF DECIMAL MIXED NUMBERS.

168. *1.* Divide 5.12 by .8.

ANALYSIS.—Regard both numbers as integers; then 512÷8=64. Dividing the divisor, 8, by 10, multiplies the quotient by 10; that is, 512÷.8=640.

Dividing the dividend, 512, by 100, divides the quotient by 100; that is, 5.12÷.8=6.40, or 6.4.

Observe that the number of decimal orders in the divisor and quotient equals the number of decimal orders in the dividend.

2. Divide 28.4967 by 5.84.

PROCESSES.

```
5.84)28.4967(4.87+
     2336
     ----
      5136
      4672
      ----
       4647
       4088
       ----
        559

5.84)28.4967(4.8795+
     2336
     ----
      5136
      4672
      ----
       4647
       4088
       ----
        5590
        5256
        ----
         3340
         2920
         ----
          420
```

SOLUTION.—Since there are two decimal orders in the divisor, and four in the dividend, there will be two decimal orders in the quotient. The remainder is 559. If the division is continued by annexing 0's to the dividend, there will be one decimal figure in the quotient for every 0 annexed to the dividend.

NOTE I.—Annexing 0 to a remainder is equivalent to annexing it to the dividend, and then "bringing it down."

NOTE II.—Incomplete division is usually indicated by annexing to the quotient the sign +.

3. Divide .48756 by 324.5.

PROCESS.

```
324.5).48756(.0015+
       3245
       ----
       16306
       16225
       -----
          81
```

SOLUTION.—Since there are five decimal orders in the dividend, and one in the divisor, there must be four in the quotient. But since the quotient contains but two figures, 15, it is necessary to prefix (not annex) two 0's.

Rule.—*Divide as in integers. Make as many decimal orders in the quotient as the number of decimal orders in the*

dividend exceeds the number of decimal orders in the divisor. If necessary, prefix 0's to the significant figures of the quotient.

SUGGESTIONS.—*If the decimal orders in divisor and dividend are equal in number, the quotient is an integer, mixed number or fraction.*

If the decimal orders in the dividend are less in number than those in the divisor, make them equal by annexing 0's to the dividend.

Before dividing, ascertain how many decimal places there should be in the quotient. When the last figure of the dividend has been used, place the decimal point in the quotient, prefixing 0's if necessary. Continue the division, if desirable, by annexing 0's to the dividend.

Divide:

4. 846.75 by 4.5.
5. 73.484 by 3.28.
6. 10.0946 by .23.
7. 64.756 by .834.
8. 7.5683 by 1.34.
9. 946.75 by 3.475.
10. 83.7456 by .845.
11. 5434.96 by 82.451.
12. 723.45 by .3464.
13. .95643 by 78.4.
14. 75678. by .0375.
15. 95.698 by .0675.
16. 3.6482 by .00045.
17. 45.836 by 2.3049.

18. Divide 438.467 by 100.

SOLUTION.—Remove the decimal point two places to the left. $438.467 \div 100 = 4.38467$.

Divide:

19. 348.75 by 100; by 1000; by 10000.
20. 75096.1 by 100; by 1000; by 10000.
21. .8 by 10; by 100; by 1000.

22. If a locomotive run 62.5 miles in 2.5 hours, what is its speed per hour?

23. What is the cost of one acre of land, if 19.5 acres cost $73.125?

24. What number, multiplied by 28.75, will produce 672.75?

25. If a ship sail 431.25 miles in 37.5 hours, what is her average speed per hour?

26. Find value of one ox, if 374 oxen are worth $11921.25.

27. One degree of latitude is about 69.16 miles. How many degrees of latitude are there in 24897.6 miles?

REVIEW QUESTIONS.

How are decimals written for addition and subtraction? What is the unit in United States money? What part of a dollar is a cent? What part of a dollar is a mill? How many decimal orders in the product? How many decimal orders in the quotient? How may the correctness of the pointing in the quotient be proved? When the decimal orders in the divisor and dividend are equal in number, what is the quotient? What should be done when the decimal orders of the dividend are less than those in the divisor? What, when the decimal orders in the dividend exceed those in the divisor?

MISCELLANEOUS PROBLEMS.

1. Add fifteen thousandths, thirty-one hundredths, one thousand and twenty-nine millionths, eighty-one ten-thousandths, three hundred and twenty-seven, seven tenths.

2. Add $84.75, $3.87½, $40.09, $.75, $.08, $5.33, $196.45, $33.62½.

3. From 9 times 345.67 subtract 984.64 divided by 8.

4. $(434.55 \div 5) + (4.347 \times .009) - (2.52 \div .03) = ?$

5. Reduce to decimals and find sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{4}{5}$, $\frac{1}{20}$, $\frac{9}{40}$.

6. Reduce to common fractions in lowest terms: .6, .375, .0625.

7. Divide 846.7896 by .06.

8. Divide 925. by .00025.

9. Divide .375 by 125.

10. Multiply 98.34 by .0078.

11. A man earned $16.25 one week, $25.62½ the next week, $18.00 the third week, and $21.37½ the fourth week. His expenses averaged $12.25 per week. How much money did he save in the 4 weeks?

12. An agent traveled 324.8 miles in January, 496.6 in February, 134.84 in March, 740.38 in April. How many miles did he average per month?

13. Mr. C. had a farm of 1800 acres. He sold 134.75 acres to Mr. D., 86.5 to Mr. E., 240.08 acres to Mr. F., and half of the remainder to Mr. G. How much remained?

14. If a ship averages 135.66⅔ miles per day, how many miles will it sail in 2 weeks and 4 days?

15. Find cost of shingles for 6 houses, if each house needs 7500 shingles, worth $4.25 per M.

16. Find cost of 22480 bricks at $9.25 per M.

17. How many feet of boards, at $12.50 per M., can be bought for $862.50?

18. How many bricks, worth $8.625 per M., can be bought for $603.75?

19. How many bushels of oats, worth .37½ per bushel, are equal in value to 62.5 bushels wheat, worth $1.12½ per bushel?

20. Divide 84.35 by 52.7185; multiply the quotient by .003. Find sum of the two results.

21. Mr. C.'s expenses were $80.75 in January, $75.375 in February, $64.625 in March, $92 in April. What did his expenses average per month? What were his expenses for a year, at the same rate?

22. What is the value of a farm if .15 of it be worth $2250?

23. 270 is ⅜ of what number?

24. 270 ÷ 1.12½ = ?

25. 270 is 1.12½ of what number?

26. 500 is 1.25 of what number?

27. A merchant sold some dry goods for $525. which was 1.05 of the cost. What was the cost?

28. Mr. D. sold an engine for $9405, which was 1.37½ of its cost. What was the cost of the engine?

29. 620 is ⅝ of what number?

30. 620 is ⅚ of what number?

31. $1440 \div .75 = ?$

32. Mr. E. sold a house for \$2880, which was .75 of its cost. What did the house cost?

33. Mr. F. sold a farm for \$7850, which was $.87\frac{1}{2}$ of its cost. What was the cost?

34. Find 32 hundredths of 284.75.

35. Find 124 hundredths of 54678.

36. $(48.75 \times .15) + (65.78 \times .75) = ?$

37. Mr. G. bought oxen for \$3150; he sold them for \$2800, and thereby lost \$5 on each ox. How many oxen did he buy?

38. Mr. H. sold 324 sheep for \$1134, and thereby gained \$1.50 on each. What did he give for them?

39. Mr. J.'s house cost \$4500, which was 3 times the cost of his furniture, and twice the cost of his lot. Find cost of all.

40. At \$5.75 per cord, how many cords of wood may be exchanged for $287\frac{1}{2}$ bushels of oats worth $37\frac{1}{2}$ cents per bushel?

41. A man gave 324.75 bushels of potatoes in exchange for 30 calves, valued at \$129.90. What did each calf cost, and what was the value of the potatoes per bushel?

42. $1.25 \times \frac{2}{5} \div 2.5 = ?$

43. $\frac{3}{8} \times .375 \div \frac{9}{64} = ?$

44. A merchant sold 5 bales of cotton at \$17.75 per hundred pounds; one bale contained 452.85 lb., the others 324.92 lb. each. What did he receive for them all?

45. Find four equal numbers the sum of which is 324.125.

46. The sum of two numbers is 535.75; one of them is 228.496. What is the other?

47. The difference of two numbers is 8.25; the less number is four times the difference. What is the sum of the two numbers?

48. The product of three numbers is 39.33; one of the numbers is 13.8; another is 7.125. What is the third number?

SECTION XI.

DENOMINATE NUMBERS.

Art. 169. Numbers are either ***Abstract*** or ***Denominate.***

170. An ***Abstract Number*** is a number whose kind of unit is not named.

171. A ***Denominate Number*** is a number whose kind of unit is named.

Thus, 1, 3, 10, are abstract numbers; 1 dollar, 3 mills, 10 horses, are denominate numbers.

172. A ***Simple Denominate Number*** expresses units of but *one* denomination.

Thus, 5 cents, 2 quarts.

173. A ***Compound Denominate Number*** expresses units of two or more denominations of the same scale.

Thus, 2 feet 3 inches; 8 pounds 10 ounces.

174. A ***Solid*** is a body which has three dimensions, length, breadth and thickness.

A book is a solid; so is a marble.

175. A ***Surface*** has two dimensions, length and breadth.

This *leaf* is a solid, because it has length, breadth and thickness. This *page* (one side of the leaf) is a surface, because it has length and breadth, but not thickness.

176. A ***Line*** has one dimension only, length.

A line is the measure of distance. This page is 8 inches long. The lines drawn on the black-board have both breadth and thickness; but they are not *mathematical* lines. Even the lines drawn by the finest lead pencil have breadth and thickness. They must have breadth, in order to be seen: but the essential part of a line is its *length.*

Measures of Length.

177. Linear measures are used in measuring distances. The dimensions of surfaces and solids are found by measuring the distances between certain points. The length of a brick is the distance from end to end.

TABLE OF LINEAR (OR LONG) MEASURE.

12 inches (in.)	=	1 foot (ft.)
3 feet	=	1 yard (yd.)
$5\frac{1}{2}$ yards, $16\frac{1}{2}$ feet	=	1 rod (rd.)
320 rods	=	1 mile (m.)

ORAL.

1. How many inches in 3 feet?

Analysis.—In 3 feet there are 3 times as many inches as in 1 foot. In 1 foot there are 12 inches; in 3 feet there are 3 times 12 inches, or 36 inches.

2. How many inches in 2 feet? In 4 feet? In 7 feet? In 8 feet?

3. How many feet in 3 yards? In 5 yards? In 6 yards?

4. How many rods in 2 miles? In 3 miles?

5. How many feet in 36 inches?

Analysis.—Since there are 12 inches in 1 foot, in 36 inches there are as many feet as 12 are contained times in 36, or 3 feet.

6. How many feet in 24 inches? In 72 inches? In 96 inches? In 30 inches? In 50 inches?

7. How many yards in 6 feet? In 18 feet? In 20 feet? In 25 feet?

8. How many inches in 2 yards?

SUGGESTION.—Find the number of inches in 1 foot; in 1 yard; in 2 yards.

9. What is the cost of 4 yards of ribbon, worth 6 cents per yard?

ANALYSIS.— 4 yards cost 4 times as much as 1 yard. 1 yard costs 6 cents; 4 yards will cost 4 times 6 cents, or 24 cents.

10. Find cost of 8 yards of muslin, at 10 cents per yd.

11. What is the cost of 7 yards of cambric, at 12 cents per yd.?

12. How many inches in $\frac{1}{2}$ of a foot?

ANALYSIS.—In one half a foot there are one half as many inches as in one foot. In one foot there are 12 inches; in $\frac{1}{2}$ of a foot there are $\frac{1}{2}$ of 12 inches, or 6 inches.

13. How many inches in $\frac{1}{4}$ of a foot? In $\frac{1}{3}$ of a foot?

14. How many rods in $\frac{1}{2}$ of a mile? In $\frac{1}{8}$ of a mile?

15. How many inches in $\frac{3}{4}$ of a foot? In $\frac{2}{3}$ of a foot?

SUGGESTIONS.—There are 3 times as many inches in $\frac{3}{4}$ of a foot as there are in $\frac{1}{4}$ of a foot.

16. How many inches in $1\frac{1}{2}$ feet? In $3\frac{2}{3}$ feet?

17. How many feet in $2\frac{1}{3}$ yards? In $5\frac{2}{3}$ yards?

18. What part of a foot is 1 inch? 6 inches? 4 inches?

19. What part of a yard is 1 foot? 2 feet?

20. What part of a yard is 1 inch? 6 inches? 9 inches?

WRITTEN.

21. How many inches in 400 yards?

SUGGESTIONS.—Reduce to feet; then to inches; or, reduce to inches at one operation, thus: Since there are 36 inches in 1 yard, in 400 yards there are 400 times 36 inches; $36 \times 400 = 14400$ inches.

22. How many inches in 328 yards? In 540 yards?

23. How many feet in 84 rods? In 250 rods?

24. How many yards in 3 miles? In 20 miles?

25. How many inches in 50 rods? In 160 rods?

26. What part of a mile is 1 yard?

27. What part of a rod is 1 inch?

28. What part of a mile are 20 inches? 5 feet?

29. Find cost of 256 yards cloth at \$3.50 per yd.

30. Find cost of 596 feet of rope, at $2\frac{1}{4}$ cents per ft.

31. How many miles can a horse travel in 12 hours, if he travels $3\frac{3}{4}$ miles in one hour?

32. A locomotive ran 512 miles in 16 hours. What was its speed per hour?

Measures of Surface.

178. A ***Plane Surface*** is a surface which does not change its direction.

A slate is a plane surface; floors and ceilings are plane surfaces. The surface of a ball is not a plane surface, because the surface constantly changes its direction.

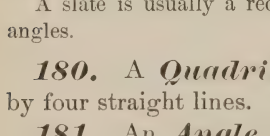
179. A ***Plane Figure*** is a portion of a plane surface.

A pane of glass in a window is a plane figure. If the sides of the pane are four straight lines, it is a *quadrilateral*. The corners of the pane of glass are called *angles*. If the pane has four corners, or angles, and they are equal to each other, they are *right angles*, and the pane is a *rectangle*.

A slate is usually a rectangle; its corners, or angles, are right angles.

180. A ***Quadrilateral*** is a plane figure bounded by four straight lines.

181. An ***Angle*** is the divergence of two straight lines which meet or intersect.



The angle A C B is formed by the meeting of the lines A C and B C at the point C.

182. A ***Right Angle*** is one of the four equal angles formed by the intersection of two straight lines.

A right angle is often one of the two equal angles formed by one straight line meeting another straight line.

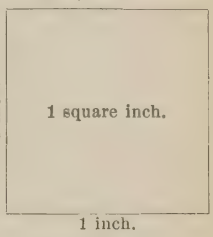
The four angles at the point 0 are right angles.

183. A ***Rectangle*** is a quadrilateral whose angles are right angles.

When the sides of a rectangle are equal, the rectangle is a *square.*

184. A ***Square*** is a rectangle having equal sides.

A square inch is a square, one inch long and one inch wide.



185. Surface, or area, is measured by square units.

A piece of paper which may be cut, without waste, into three pieces, each one square inch in size, contains three square inches.

The area of floors and walls may be expressed in square feet or in square yards.

Farms are usually described as containing a certain number of *acres;* as 80 acres, 160 acres, etc.

Counties, states, and other large portions of the earth's surface, are measured by square *miles.*

186. Area is considered a product, of which length and breadth are the factors. The number of square inches in a square foot is the product of the length and breadth of the square foot, expressed in inches: $12 \times 12 = 144$.

187. TABLE OF SQUARE MEASURE.

144	square inches	(sq. in.)	=	1	square foot	(sq. ft.)
9	square feet	-	=	1	square yard	(sq. yd.)
$30\frac{1}{4}$	square yards	-	=	1	square rod	(sq. rd.)
160	square rods	-	=	1	acre	(A.)
640	acres	-	=	1	square mile	(sq. m.)

ORAL.

1. How many square inches in 2 sq. ft.?

2. How many square yards in 54 sq. ft.?

3. How many square feet in 4 sq. rds.?

4. How many square inches in $\frac{1}{4}$ of a sq. ft. In $\frac{1}{2}$ of a sq. ft.? In $\frac{3}{4}$ of a sq. ft.?

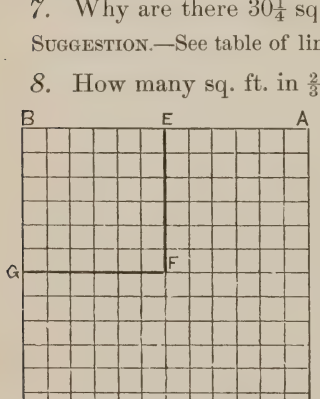
5. How many sq. rods in $\frac{1}{4}$ of an acre?

6. How many acres in $\frac{1}{4}$ of a sq. mile?

7. Why are there $30\frac{1}{4}$ square yards in a square rod?

SUGGESTION.—See table of linear measure.

8. How many sq. ft. in $\frac{2}{3}$ of a sq. yd.?



NOTE I.—This diagram illustrates the fact that the square of 12 is 144; the square of 12 *linear* inches is 144 *square* inches.

NOTE II.—The line A E is *one half* of the line A B; but the square A E F G is *one fourth* of the square A B C D. That is, the square of *one half* of a line is *one fourth* of the square of the whole line. So, the square of $\frac{1}{3}$ is $\frac{1}{9}$; etc. See Art. 143.

9. How many inches in $\frac{1}{4}$ of a sq. ft.?

10. How many inches in the square of $\frac{1}{2}$ of a foot?

11. A man having a field containing 144 sq. rods sold 36 sq. rods. What part of his field did he sell?

12. What part of a sq. yd. is a sq. ft.?

13. What part of a square ft. are 36 sq. inches? 72 sq. inches? 108 sq. inches?

14. What part of a sq. foot is the square of $\frac{1}{2}$ of a foot?

15. What part of a sq. foot is the square of 3 inches?

16. What part of a sq. foot is the square of $\frac{1}{3}$ of a foot?

17. What part of a square mile are 160 sq. rods?

WRITTEN.

18. How many sq. inches in 29 sq. ft.?

19. How many sq. ft. in 9324 sq. yds.?

20. How many sq. ft. 43200 sq. inches?

21. How many sq. yards in 3042 sq. feet?

22. How many sq. miles in 3200 acres?

23. How many acres in 329 sq. miles?

24. How many sq. yards in 5 acres?

25. How many sq. rods in 16 miles?

26. How many sq. rods in 588060 sq. ft.?

27. How many acres in 2178000 sq. ft.?

28. How many sq. inches in 1 acre?

29. What part of 1 sq. mile is 1 sq. rod?

30. What part of 1 sq. mile are 25 sq. rods?

31. What part of 1 sq. yd. is 1 sq. inch?

32. What part of 1 sq. yd. is the sq. of $\frac{1}{2}$ of a sq. ft.?

33. Draw a diagram illustrating the difference between 9 square feet and 9 feet square.

Angular or Circular Measure.

188. A ***Circle*** is a plane figure bounded by a line every point of which is equally distant from a point within the circle, called the *center*.

189. A ***Circumference*** is the line which bounds a circle.

190. An ***Arc*** is any part of a circumference.

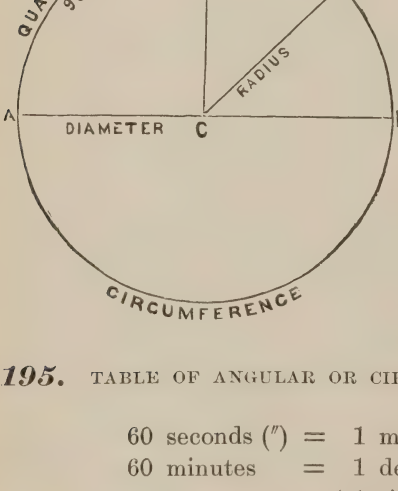
191. A ***Degree*** is one of the 360 equal parts of a circumference.

Every circumference contains 360 degrees; the size of a degree, therefore, depends upon the size of the circumference of which it is a part.

192. A ***Diameter*** is a straight line extending through the center of a circle, terminating in opposite points of its circumference.

193. A ***Radius*** is a straight line extending from the center of a circle to any point in its circumference. A *radius* is one half of a *diameter* of the same circle.

194. An arc measures the angle formed by two radii drawn to its extremities.



The arc D E measures the angle D C E. Each is 45 degrees.

195. TABLE OF ANGULAR OR CIRCULAR MEASURE.

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
90 degrees	=	1 right angle. 1 quadrant.
360 degrees	=	1 circumference (cirf.)

NOTE.—A degree on the equator is $69\frac{1}{6}$ miles.

ORAL.

1. How many seconds in 2 minutes? In 3 minutes? In 5 minutes?

2. How many minutes in 4 degrees? In 6 degrees? In 8 degrees? In 10 degrees?

3. How many seconds in $\frac{1}{2}$ of a minute? In $\frac{1}{4}$ of a minute? In $\frac{1}{12}$ of a minute?

4. How many minutes in $\frac{3}{4}$ of a degree? In $\frac{2}{3}$ of a degree? In $\frac{4}{5}$ of a degree?

5. How many minutes in $2\frac{1}{2}$ degrees? $3\frac{1}{3}$ degrees? $5\frac{2}{5}$ degrees.

6. How many minutes in 120″? In 360″? In 480″?

7. How many degrees in 240′? In 180′? In 720′?

8. How many degrees in 3600′?

9. How many degrees in 1 right angle? In 2 right angles?

10. How many quadrants are equal to 3 right angles?

11. How many quadrants in a circumference?

12. How many right angles at the center of a circle?

13. How many degrees in $3\frac{1}{3}$ right angles?

14. How many degrees in one sixth of a circumference?

NOTE.—One sixth of a circumference is called a **Sextant.**

15. How many degrees in 2 sextants?

16. How many quadrants are equal to 3 sextants?

17. How many right angles in $4\frac{1}{2}$ sextants?

18. How many degrees in one twelfth of a circumference?

NOTE.—One twelfth of a circumference is called a **Sign.**

19. How many degrees in 2 signs? In 3 signs? In 5 signs?

20. How many signs in 1 circumference? In 1 sextant? In 1 quadrant?

21. How many signs in 180°? How many sextants? How many quadrants? How many right angles?

WRITTEN.

22. How many degrees in 320′?

23. How many minutes in 480″?

24. How many degrees in 7200″?

25. How many seconds in $5\frac{1}{2}$ degrees?

26. How many seconds in one circumference?

27. How many degrees in 28800″?

28. A degree of the equator is $69\frac{1}{6}$ miles. What is the length of the equator in miles?

29. What is a quadrant of the earth's equatorial circumference?

30. A ship has sailed 129 degrees. How many miles has she sailed, if each degree equals $54\frac{2}{3}$ miles?

31. Thirty degrees of a hoop measured 60 inches. What was the length of the hoop in feet?

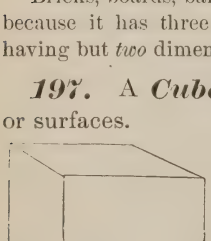
32. If a quadrant, a sextant, and a sign, be taken from a circle, how many degrees will remain?

Measures of Volume.

196. A ***Solid*** is a body which has three dimensions, length, breadth, and thickness.

Bricks, boards, balls, stones, etc., are solids. Water is called a solid, because it has three dimensions; the *surface* of water is a plane, having but *two* dimensions.

197. A ***Cube*** is a solid having six equal square sides or surfaces.



A cubic inch is a solid 1 inch long, 1 inch wide, 1 inch thick. It has six surfaces, each 1 inch square. It has 12 *edges*, each 1 inch long.

198. Solidity, or volume, is measured by units of volume,

or cubical units. A room is said to contain a certain number of cubic feet; a gallon contains 231 cubic inches; etc.

Volume is a product, of which length, breadth, and thickness, are the factors. The number of cubic inches in a cubic foot is the product of the length, breadth, and thickness of a cubic foot expressed in inches: $12 \times 12 \times 12 = 1728$.

199. TABLE OF CUBIC MEASURE.

1728	cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27	cubic feet	= 1 cubic yard (cu. yd.)
$24\frac{3}{4}$	cubic feet	= 1 perch of stone or masonry.
128	cubic feet	= 1 cord (cd.)

ORAL.

1. How many cubic feet in 2 cubic yards?
2. How many cubic inches in $\frac{1}{2}$ of a cu. ft.?
3. How many cubic feet in $\frac{1}{3}$ of a perch of stone?
4. How many cubic feet in $\frac{1}{8}$ of cord?

WRITTEN.

5. How many cu. in. in 36 cu. ft.?
6. How many cu. feet in 691200 cu. in.?
7. How many cu. feet in 428 cords?
8. How many cords in 64000 cu. ft.?
9. How many cu. ft. in 40 perch of stone?
10. How many perch of masonry in 24750 cu. ft.?
11. How many cu. inches in 1 cord?
12. What part of a cu. foot is a cu. inch?
13. What part of a cord is 1 cu. yard?
14. What part of 5 cords are 12 cu. ft.?

200. Liquid measures are used in measuring liquids.

TABLE OF LIQUID MEASURE.

4 gills (gi.)	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts	=	1 gallon (gal.)
31½ gallons	=	1 barrel (bbl.)
63 gallons (2 bbls.)	=	1 hogshead (hhd.)

NOTE.— 1 gallon=231 cubic inches.

ORAL.

1. How many gills in 7 pints? In 10 pints? In 20 pints?

2. How many pints in three quarts? In 5 quarts? In 8 quarts?

3. How many quarts in 5 gallons? 9 gallons? 12 gallons?

4. How many pints in 40 gills? In 80 gills? In 100 gills?

5. How many quarts in 24 pints? In 36 pints?

6. How many gallons in 16 quarts? In 44 quarts? In 60 quarts?

7. How many gallons in 2 barrels? In 10 barrels?

8. How many quarts in one barrel?

9. How many barrels in 7 hogsheads? In 9 hogsheads?

10. How many hogsheads in 12 barrels? In 30 barrels?

11. How many cubic inches in 1 gallon? In 2 gallons?

WRITTEN.

12. How many pints in 3460 gills?

13. How many quarts in 1824 pints?

14. How many gallons in 8 barrels?

15. How many quarts in 2 barrels?

16. How many pints in 327 quarts?

17. How many pints in 10 hogsheads?

SUGGESTIONS.—First find the number of gallons in 10 hogsheads; then change the gallons to quarts, etc.

18. How many gills in 20 barrels?
19. How many gills in 3 hogsheads?
20. How many cubic inches in 8 gallons?
21. How many cubic inches in 1 barrel?

201. Dry measures are used in measuring grain, fruit, etc.

TABLE OF DRY MEASURE.

2 pints (pt.)	=	1 quart (qt.)
8 quarts	=	1 peck (pk.)
4 pecks	=	1 bushel (bu.)

NOTE.— 1 bushel=2150.4 cubic inches.

ORAL.

1. How many pints in 16 quarts? In 25 quarts?
2. How many quarts in 40 pints? In 48 pints?
3. How many quarts in 3 pecks? In 8 pecks?
4. How many pecks in 32 quarts? In 40 quarts?
5. How many pecks in 5 bushels? In 7 bushels?
6. How many bushels in 24 pecks? In 36 pecks?
7. What part of a peck is a quart?
8. What part of a bushel is a peck?
9. How many quarts in 1 bushel?
10. What part of a bushel is 1 quart? 3 quarts?

WRITTEN.

11. How many quarts in 329 pecks?
12. How many bushels in 444 pecks?
13. How many quarts in 72 bushels?
14. How many bushels in 12800 quarts?
15. How many pecks in 6400 pints?
16. What part of a bushel is 1 quart?
17. What part of a bushel are 3 pints?
18. How many cubic inches in 3 bushels?
19. How many cubic inches in 1 peck?
20. How many bushels in 21504 cubic inches?

Measures of Weight.

202. Avoirdupois weight is used in weighing merchandise that is bought and sold by weight, such as groceries, coal, etc.

TABLE OF AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	=	1 pound (lb.)
100 pounds	=	1 hundredweight (cwt.)
20 hundredweight	=	1 ton (T.)
2000 pounds	=	

NOTE.—The "long ton," containing 2240 lb., is used in Great Britain, in United States Custom Houses, and in Pennsylvania coal mines.

ORAL.

1. How many ounces in 3 pounds? In 5 pounds?
2. How many pounds in 32 ounces? In 64 ounces?
3. How many ounces in 1 cwt.? In 2 cwt.?
4. How many pounds in half a ton?
5. How many pounds in a quarter of a cwt.?
6. How many cwt. in one tenth of a ton?
7. What part of a pound are 5 ounces?
8. What part of a cwt. is one ounce?

WRITTEN.

9. How many lb. in 17 tons?
10. How many cwt. in 25600 ounces?
11. How many tons in 598000 lb.?
12. How many ounces in 3240 lb.?
13. How many lb. in 38 cwt.?
14. How many oz. in 2 T.?
15. What part of 1 cwt. is 1 oz.?
16. What part of 1 T. is 1 lb.?

203. Troy weight is used for weighing gold, silver, and precious stones.

TABLE OF TROY WEIGHT.

24 grains (gr.)	=	1 pennyweight (pwt.)
20 pennyweights	=	1 ounce (oz.)
12 ounces	=	1 pound (lb.)

ORAL.

1. How many grains in 2 pwt.?
2. How many pwt. in 72 grains?
3. How many oz. in 5 lb.? In 7 lb.?
4. How many oz. in 100 pwt.? In 140 pwt.?

WRITTEN.

5. How many pwt. in 18 lb.?
6. How many gr. in 15 oz.?
7. How many pwt. in 9600 gr.?
8. How many lb. in 14400 pwt.?
9. What part of a lb. is 1 pwt.?
10. What is the value of 9 oz. of pure gold, if 1 oz. is worth $20.672?
11. What is the value of 7 lb. of pure silver, if 1 oz. is worth $1.388?
12. A gold dollar weighs 1 pwt. What is the value of a bag of gold dollars weighing 5 lb.?
13. What is the value of 1 gr. of pure gold?

See example 10.

14. What is the difference in value between 5 oz. of pure gold and 5 lb. of pure silver?

204. Apothecaries' weight is used in compounding prescriptions.

TABLE OF APOTHECARIES' WEIGHT.

20 grains (gr.)	=	1 scruple (sc. or ℈.)
3 scruples	=	1 dram (dr. or ʒ.)
8 drams	=	1 ounce (oz. or ℥.)
12 ounces	=	1 pound (lb. or ℔.)

ORAL.

1. How many gr. in 5 sc.? In 8 sc.? In $3\frac{1}{2}$ sc.?
2. How many dr. in 18 sc. In 60 sc.? In 90 sc.?
3. How many oz. in 96 dr.? In 40 dr.? In 60 dr.?
4. How many oz. in 20 lb.? In 30 lb.? In 40 lb.?
5. How many lb. in 24 oz.? In 36 oz.? In 72 oz.?

WRITTEN.

6. How many gr. in 8 scruples?
7. How many sc. in 17 lb.?
8. How many drams in 7200 gr.?
9. How many ounces in 200 prescriptions, each weighing 3 grains?
10. How many lb. in 28800 gr.?

205. Measures of Time.

TABLE OF TIME.

60 seconds (sec.)	=	1 minute (m.)
60 minutes	=	1 hour (h.)
24 hours	=	1 day (d.)
7 days	=	1 week (wk.)
30 days	=	1 month (mo.)
365 days	=	1 year (yr.)
12 months	=	
52 weeks 1 day	=	
100 years	=	1 century (C.)

NOTE I.—A month of 30 days is called a business month. The actual number of days in the different months of the year varies from 28 to 31, as may be seen in the table below.

NOTE II.—Leap year has 366 days.

NOTE III.—Every year whose number is divisible by 4 is a leap year, unless its number ends with a double cipher, in which case its number must be divisible by 400. Thus, 1872, 1876, etc., are leap years; 1875, 1878, etc., are not leap years. 1600 and 2000 are leap years; 1800 and 1900 are not leap years.

The year is divided into seasons and months as follows:

Seasons.		Months.	Abbreviations.	No. of Days.	
Winter	1.	January.	Jan.	31.	
	2.	February.	Feb.	28.	In leap year 29.
Spring	3.	March.	Mar.	31.	
	4.	April.	Apr.	30.	
	5.	May.	—	31.	
Summer	6.	June.	—	30.	
	7.	July.	—	31.	
	8.	August.	Aug.	31.	
Autumn	9.	September.	Sept.	30.	
	10.	October.	Oct.	31.	
	11.	November.	Nov.	30.	
Winter	12.	December.	Dec.	31.	

ORAL.

1. How many minutes in a quarter of an hour? In 3 quarters? In half an hour?

2. How many minutes in 2 hours? In 5 hours?

3. How many days in 7 weeks? In 10 weeks?

4. How many weeks in 21 days? In 35 days?

5. How many years in 60 months?

6. How many months in 4 years?

7. How many years in a quarter of a century?

8. How many weeks in February, 1875?

9. How many days in February, 1876?

10. How many days from March 10 to May 30?

11. What o'clock is 6 hours after 9 o'clock A.M.?

12. What day of the week is 2 hours before 1 o'clock A.M. on Wednesday?

WRITTEN.

13. How many seconds in 5 hours?

14. How many days in 1440 hours?

15. How many hours in 5400 minutes?

16. How many weeks in 1696 days?

17. How many years in 25 centuries?
18. How many days in 10 years, including 3 leap years?
19. How many days in the last six months of the year?
20. What date is 90 days after May 13?
21. What date is 3 yr. 5 mo. 20 d. after April 1, 1875?

206. PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
10 reams = 1 bale.

ORAL.

1. How many sheets in 2 quires? In 4 quires?
2. How many sheets in $\frac{1}{2}$ quire? In $\frac{3}{4}$ quire?
3. How many quires in 72 sheets? In 240 sheets?
4. How many quires in 3 reams? In 5 reams?
5. How many reams in 4 bales? In 6 bales?
6. How many bales in 30 reams? In 60 reams?
7. How many sheets in 1 ream? In $\frac{1}{2}$ ream?
8. How many quires in $\frac{1}{2}$ ream? How many sheets in $\frac{1}{4}$ ream?

WRITTEN.

9. How many quires in 14400 sheets?
10. How many reams in 600 quires?
11. How many sheets in 1 bale?
12. How many bales in 540 reams?
13. How many quires in 25 bales?

207. MISCELLANEOUS.

12 units = 1 dozen (doz.)
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

ORAL.

1. How many units in 5 dozen? In $3\frac{1}{2}$ dozen?
2. How many dozen in 72 units? In 108 units?
3. How many units in a gross?
4. How many dozen in a great gross?
5. What part of a gross are 72 units?
6. How many units in 3 score? In 8 score?
7. How many score in 100 units? In 140 units?

WRITTEN.

8. How many units in 1 great gross?
9. How many units in 35 score?
10. How many gross in 720 dozen?
11. How many dozen in 527 great gross?
12. How many dozen in 172800 units?
13. What part of a great gross is a unit?
14. What part of a score is a dozen?
15. What part of a gross is a score?
16. How many score in 10 dozen?
17. How many gross in 36 score?

REVIEW QUESTIONS.

Into what two classes may numbers be divided in regard to the unit? Define abstract number. Define denominate number. Define simple denominate number. Define compound denominate number. What is a solid? A surface? A line? For what are linear measures used? Repeat the table. What is a plane surface? What is a plane figure? A quadrilateral? What is an angle? A right angle? A rectangle? A square? How is surface measured? Repeat the table. How may area or surface be considered? What is a circle? A circumference? An arc? A degree? A diameter? A radius? By what is an angle measured? Repeat the table of angular measure. What is a sextant? A sign? What is a solid? A cube? How is solidity or volume measured? Of what may volume be considered the product? Repeat the table of cubic measure. Of liquid measure. How many cubic inches in a gallon? Repeat table of dry measure.

How many cubic inches in a bushel? How is avoirdupois weight used? Repeat the table. Where it the long ton used? How does it differ from the common ton? How is Troy weight used? Repeat the table. How is apothecaries' weight used? Repeat the table. Repeat the table of time. How many days in leap year? How may leap year be known? Repeat the table of paper. The miscellaneous table.

REDUCTION OF DENOMINATE NUMBERS.

208. The ***Reduction*** of a denominate number is a change of its expression without a change of its value.

Thus, 3 feet may be changed to 1 yard; 1 foot 6 inches may be changed to 18 inches, or to $\frac{1}{2}$ yard.

209. ***Reduction Descending*** is the change from a higher to a lower denomination.

1 gal.=4 qts.

210. ***Reduction Ascending*** is the change from a lower to a higher denomination.

4 qts.=1 gal.

Reduction Descending.

211. *1.* How many inches in 5 yd. 2 ft. 8 in.?

Process.

5 yd. 2 ft. 8 in.
3

15 ft.
2 ft.

17 ft.
12

204 in.
8 in.

212 in.

Outline of Analysis.—Reduce the 5 yd. to ft.; add the 2 ft.; reduce the ft. to inches; add the 8 inches.

Analysis.— 1. In 1 yd. there are 3 ft.; in 5 yds. there are 5 times 3 ft., or 15 ft.

2. 15 ft.+2 ft.=17 ft.

3. In 1 ft. there are 12 inches; in 17 ft. there are 17 times 12 inches, or 204 inches.

4. 204 in.+8 in.=212 in. *Ans.*

Rule for Reduction Descending.

I. *Use as factors the number of the highest denomination given, and the value of one of that denomination in units of the next lower denomination. The product will be units of the lower denomination.*

II. *To this product add the given number, if any, of that denomination.*

III. *Make this sum a factor of another product* (*as in* I), *and so proceed, until the numbers are reduced to the required denomination.*

2. How many seconds in 1 h. 15 m. 30 sec.?

3. How many sq. ft. in 25 sq. yd. 7 sq. feet?

4. Change 4 bu. 3 pk. 2 qt. 1 pt. to pints.

5. Reduce 2 T. 15 cwt. 25 lb. to lb.

6. Reduce 3 bbl. 20 gal. to qt.

7. How many cu. ft. in $4\frac{1}{2}$ cords?

8. Change 3° 30′ 40″ to seconds of time.

9. How many sheets in 2 reams, 10 quires, 12 sheets?

10. How many units in 1 great gross, 2 gross, 5 dozen?

11. Reduce 2 weeks to minutes.

12. Reduce a leap year to hours.

13. How many years in $\frac{3}{4}$ of a century?

14. Reduce 18 cwt. 60 lb. 12 oz. to oz.

15. Reduce 1 T. 80 lb. to oz.

16. How many grains in 5 lb. 3 oz. 4 pwt. of gold?

17. How many scruples in 10 lb. 7 dr. 2 sc., apothecaries' weight?

18. Reduce 2 bu. 6 qt. to pints.

19. Reduce 5 hhd. to qt.

20. Reduce 8 bbl. 16 gal. 2 qt. 1 pt. 1 gi. to gi.

21. How many cu. in. in 8 cu. yd. 4 cu. ft. 100 cu. in.?

22. How many seconds in 1 cirf. 60°?

23. How many sq. rd. in 3 sq. m. 320 A. 40 sq. rd.?

24. Reduce 1 m. 80 rd. 4 yd. to ft.

25. How many miles in the equator?

See Note to table of angular measure.

26. How many cu. in. in 4 gal.?

See Note to table of liquid measure.

27. How many cu. in. in 25 gal.?

28. How many cu. in. in 3 bu.?

See Note to table of dry measure.

29. How many cu. in. in 40 bu.?

30. How many cu. in. in 1 pk.?

Reduction Ascending.

212. *1.* Reduce 101 pints, dry measure, to integers of higher denominations.

Process.

2)101 pt.

8) 50 qu. 1 pt.

4) 6 pk. 2 qt.

1 bu. 2 pk.

101 pt.=1 bu. 2 pk. 2 qt. 1 pt. *Ans.*

Outline of Analysis.—Reduce pints to quarts; quarts to pecks; pecks to bushels.

Analysis.—1. There are 2 pts. in 1 qt.; in 101 pints there are as many quarts as 2 pints are contained times in 101 pints; $\frac{101}{2}$ qt.=50 qt. 1 pt.

2. There are 8 qt. in 1 pk.; in 50 qt. there are as many pecks as 8 quarts are contained times in 50 quarts; $\frac{50}{8}$ qt.=6 pk. 2 qt.

3. There are 4 pk. in 1 bu.; in 6 pk. there are as many bushels as 4 pecks are contained times in 6 pecks; $\frac{6}{4}$ bu.=1 bu. 2 pk.

Rule for Reduction Ascending.

I. *Divide the number given by the number of its denomination which equals a unit of the next higher denomination. The quotient will be the number of that denomination.*

II. *Divide this quotient by the number of its denomination which equals a unit of the next higher denomination. So proceed. The last quotient, with the several remainders, will be the result sought.*

Note.—Reduction Ascending is the converse of Reduction Descending. One is proof of the other.

2. How many ft. in 720 inches?

3. Reduce 1280 rods to miles.

4. Reduce 3729 sq. in. to sq. yd.
5. Reduce 9200 sq. rd. to sq. miles.
6. Reduce 10900″ to cirf.
7. Reduce 48960 cu. in. to cords.
8. Reduce 6400 gi. to gal.
9. Reduce 1200 pt. to bu.
10. Reduce 18246 oz. av. to integers of higher denom.
11. Reduce 5600 gr. Troy to lb. Troy.
12. Reduce 3000 gr. apoth. to lb. apoth.
13. Reduce 7200 m. to weeks.
14. Reduce 148976 h. to years.
15. Reduce 6249 sheets to bales.
16. Reduce 5897 ft. to miles.

SOLUTION.

5897 ft.÷3=1965 yd. 2 ft.
1965 yd.÷5½=3930÷11=357 rd. 1½ yd.
357 rd.÷320=1 m. 37 rd.
5897 ft.=1 m. 37 rd. 1½ yd. 2 ft.
But ½ yd.=1 ft. 6 in.; 1 ft. 6 in.+2 ft.=3 ft. 6 in.=1 yd. 0 ft. 6 in.
1 yd.+1 yd. 0 ft. 6 in.=2 yd. 0 ft. 6 in.
5897 ft.=1 m. 37 rd. 2 yd. 0 ft. 6 in. *Ans.*

17. Reduce 11221 ft. to miles.
18. Reduce 625 sq. ft. to higher denominations.

MISCELLANEOUS PROBLEMS.

ORAL.

1. Find cost of 8½ yds. of ribbon, worth 20 cents per yd.

ANALYSIS I.— 8½ yd. of ribbon cost 8½ times the cost of 1 yd. 1 yd. cost 20 cents, 8½ yd. cost 8½ times 20 cents, or $1.70.

ANALYSIS II.—Since 1 yd. cost 20 cents, 8½ yd. cost 8½ times 20 cents, or $1.70.

ANALYSIS III.— 8½ yd. of ribbon at 20 cents per yd. cost 8½ times 20 cents, which is $1.70.

NOTE.—The teacher should direct the pupil which analysis to use. Analyses should be used till the pupil understands the logic of the example; then results only

should be demanded, and these as quickly as possible. Teachers should improvise oral examples until accuracy and rapidity are secured. For other forms of analysis, see pages 72, 81 and 89.

2. Find cost of 50 feet of rope, at 4 cents per foot.

3. Find cost of 8 yd. calico, at $12\frac{1}{2}$ cents per yd.

4. Find cost of 1 rod of fence, if 7 rods cost \$175.

Analysis I.— 1 rod costs one seventh of the cost of 7 rods. 7 rods cost \$175; 1 rod costs one seventh of \$175, or \$25.

Analysis II.—Since 7 rd. cost \$175, 1 rod costs one seventh of \$175, or \$25.

Analysis III.— 1 rod costs one seventh of \$175, the cost of 7 rods, or \$25.

5. Find cost of 1 ft. of wire, if 60 feet cost \$1.20.

6. Find cost of 1 sq. yd. of carpet, if 9 sq. yd. cost \$27.

7. Find cost of 40 acres of land, at \$2.50 per acre.

8. Find cost of 7 gal. vinegar, at $12\frac{1}{2}$¢ per gal.

9. Find cost of 5 bbl. ale, at $\$9\frac{1}{4}$ per bbl.

10. Find cost of 30 bu. wheat, at \$1.50 per bu.

11. Find cost of 50 lb. sugar, at 12¢ per lb.

12. What is the value of 12 T. coal, at \$8.50 per T.?

13. What is the value of 10 oz. standard silver, worth \$1.25 per oz.?

14. What is the cost of 8 lb. coffee, if 5 lb. cost \$1.50?

Analysis.—Since 5 lb. cost \$1.50, 1 lb. cost $\frac{1}{5}$ of \$1.50, which is \$.30; and 8 lb. cost 8 times \$.30, or \$2.40.

15. What is the cost of 3 lb. sugar, if 7 lb. cost 63 cents?

16. What is the cost of 12 bu. oats, if 5 bu. cost \$2?

17. If 4 yd. of cloth cost \$16, what is the cost of 9 yd. at the same rate?

18. If 20 gal. wine are worth \$10, what is the value of 17 gal. of the same article?

19. If 15 cords of wood are worth \$45, what is the value of 11 cords?

20. Cost of 5 cwt. of flour, if 12 cwt. cost \$48?

21. Cost of 40 lb. nails, if 5 lb. cost 25 cents?

SUGGESTION.— 40 is 8 times 5.

22. Cost of 24 T. coal, if 6 T. cost $42?

23. Cost of 36 bu. oats, if 12 bu. cost $3.60?

24. Cost of 2 pk. of meal, at 6¢ per qt.?

ANALYSIS I.—At 6¢ per qt., 1 pk. costs 48¢, and 2 pk. cost 96¢.

ANALYSIS II.—Since there are 8 qt. in 1 pk., 1 pk. costs 8 times as much as 1 qt., that is, 8 times 6¢, or 48¢; and 2 pk. cost 2 times 48¢, or 96¢.

ANALYSIS III.—In 2 pk. there are 16 qt.; since 1 qt. costs 6¢, 16 qt. cost 16 times 6¢, or 96¢; that is, 2 pk. cost 96¢.

25. What is the cost of 3 bu. corn, at 10¢ per pk.?

26. What is the cost of 4 bu. wheat, at 20¢ per pk.?

27. What is the cost of 5 pk. of potatoes, at 3¢ per qt.?

28. Cost of 2 lb. cinnamon, at 20¢ per oz.?

29. Cost of 7 T. iron, at $3 per cwt.?

30. Cost of 9 yd. cord, at 3¢ per ft.?

31. Cost of 1 hhd. wine, at $2 per gal.?

32. Cost of 2 bbl. vinegar, at 40¢ per gal.?

33. Cost of $4\frac{1}{2}$ gal. milk, at 5¢ per qt.?

34. Cost of $3\frac{1}{4}$ lb. honey, at 2¢ per oz.?

35. Cost of 3 oz. sago, at 48¢ per lb.?

ANALYSIS.— 1 oz. costs $\frac{1}{16}$ of 48¢, or 3¢; 3 oz. cost 3 times 3¢, or 9¢.

36. Cost of 3 quires of paper, at $4 per ream?

37. Cost of half a dozen thimbles, worth 60¢ a gross?

38. Value of 5 oz. of gold, worth $160 per lb. Troy?

39. Value of $\frac{3}{4}$ of an acre of land, if 3 acres are worth $60?

40. Value of 80 acres of land, if a sq. mile is worth $960?

WRITTEN.

41. What is the value of 120 T. coal, worth $8.25 per T.?

42. What are 73 cords of wood worth, at $2.50 per cd.?

43. Cost of $22\frac{1}{2}$ bbl. flour, at \$6.75 per bbl.?

44. Find cost of 40 gal. molasses, worth $\$.37\frac{1}{2}$ per gal.

45. Value of 160 acres of land at \$15.75 per A.?

46. Value of 36 oz. pure gold, worth \$20.672 per oz.?

47. Value of 45 oz. standard gold, worth \$18.605 per oz.?

48. Cost of 18 cwt. of tobacco, worth \$75 per cwt.?

49. Cost of 325 bu. rye, worth $62\frac{1}{2}¢$ per bu.?

50. Cost of 70 perch of stone, at \$45 per perch?

51. What is the cost of 360 bbl. flour, if 17 bbl. cost \$102?

52. What is the cost of 37 cords of wood, if 9 cords cost \$29.25?

53. Cost of $53\frac{3}{4}$ bu. wheat, if 13 bu. cost \$16.25?

54. Cost of $18\frac{1}{2}$ T. coal, if 5 tons cost \$25?

55. Cost of 30 acres of land, if $5\frac{1}{2}$ acres cost \$33?

56. Cost of 52 reams of paper, if 26 reams cost \$78?

57. Cost of $5\frac{4}{5}$ cu. yd. of earth, if 3 cu. yd. cost \$2.25?

58. Value of $8\frac{3}{4}$ yd. cloth, if $21\frac{1}{4}$ yd. cost \$42.50?

59. Value of 24 miles of wire, if $2\frac{1}{3}$ miles cost \$42?

60. Value of 57 acres of land, if 160 A. are worth \$400?

61. Find cost 30 bu. of beans, at $\$1.12\frac{1}{2}$ per pk.

62. Find cost of 21 gal. cider, at $4\frac{1}{2}¢$ per qt.

63. Find cost of 3 T. nails, at $3¢$ per lb.

64. Find cost of 33 sq. rd. of land, at \$640 per A.

65. Find cost of 320 sq. yd. of oil-cloth, at $16¢$ per sq. ft.

66. What is the value of $1\frac{1}{2}$ sq. miles of land, at \$.30 per sq. rd.?

67. Value of 17 quires of paper, worth \$2.40 per ream?

68. Value of 5 acres of land, worth \$.50 per sq. ft.?

69. Value of 3 cu. yd. of stone, worth \$6.40 per cord?

70. Cost of 5 hhd. vinegar, worth $8¢$ per qt.?

71. Find cost of 2 yd. 1 ft. 6 in. of chain, at $50¢$ per yd.

SUGGESTION.—Reduce the ft. and in. to fractions of a yd.

72. Find cost of 4 A. 40 sq. rd. of land worth $60 per A.

73. Find cost of 6 cd. 32 cu. ft. of wood at $4 per cd.

74. Find cost of 20 perch $8\frac{1}{4}$ cu. ft. of stone, at $3.30 per perch.

75. Find cost of 400 cu. yd. 13 cu. ft. 864 cu. in. of masonry, at $1.75 per cu. yd.

76. Value of 15 bbl. 15 gal. 3 qt. spirits, worth $150 per bbl.?

77. Cost of 85 bu. 3 pk. corn, at $.50 per bu.?

78. Cost of 32 T. 2 cwt. 50 lb. of iron ore, worth $12 per T.?

79. Value of 7 oz. 15 pwt. of gold, worth $20.50 per oz.?

80. Cost of 4 gr. gross 8 gross buttons, at $2.50 per gr. gross?

ADDITION AND SUBTRACTION.

213. *1.* Find the sum of 3 yd. 1 ft. 7 in.; 2 ft. 8 in.; 1 yd. 1 ft. 10 in.

PROCESS.

yd.	ft.	in.
3	1	7
	2	8
1	1	10
6	0	1

ANALYSIS.—1. Write the numbers to be added so that units of the same denomination shall stand in the same column.

2. Find the sum of the numbers in the column of the lowest denomination (inches), which is 25 inches=2 ft. 1 in. Write the 1 in. under the column of in.; add the 2 ft. to the column of ft.

3. Find the sum of the column of ft., which is 6 ft.=2 yd. 0 ft. Write the 0 under the column of ft.; add the 2 yd. to the column of yd.

4. Find the sum of the column of yd., which is 6 yd.

NOTE.—By comparing the analysis of addition of simple numbers, page 41, it will be seen that the same principles underlie the addition of both simple and compound numbers. In the addition of simple numbers, the sum of each column is always divided by 10, the remainder written, and the quotient added to the next higher column. In the addition of compound numbers, the sum of each column is divided by the number of units which equal *one* of the next higher denomination, the remainder written, and the quotient added to the next higher column. In subtraction, multiplication and division, the same principles are applied.

Rule for Addition of Compound Numbers.

1. *Write the numbers to be added so that units of the same denomination shall stand in the same column.*

II. *Find the sum of the numbers in the column of the lowest denomination.*

III. *If this sum is less than the number required to equal a unit of the next higher denomination, write it under its own column. If the sum is greater than the number required to equal a unit of the next higher denomination, divide it by this number; write the remainder, if any, under its column, and add the quotient to the next higher column.*

IV. *So proceed. Under the highest column write its sum.*

2. Add 5 T. 8 cwt. 10 lb. 6 oz.; 2 T. 16 cwt. 45 lb. 8 oz.; 75 lb. 10 oz.

3. Add 10 bu. 1 pk. 2 qt.; 7 bu. 3 pk. 5 qt.; 20 bu. 4 qt.

4. Add 5 m. 140 rd. 3 yd.; 2 m. 10 rd. 2 ft. 5 in.; 4 yd. 1 ft. 11 in.; 10 m. 100 rd. 5 yd. 8 in.

5. Add 25° 39′ 40″; 72° 16′ 56″; 120° 45′ 17″; 28′ 34″.

6. Add 5 A. 120 sq. rd. 7 sq. ft. 129 sq. in.; 20 A. 140 sq. rd. 25 sq. yd. 8 sq. ft.; 1 sq. m. 50 A. 100 sq. rd.

7. From 18 gal. 2 qt. 1 pt. 2 gi. subtract 10 gal. 1 qt. 1 pt. 3 gi.

PROCESS.

gal.	qt.	pt.	gi.
18	2	1	2
10	1	1	3
8	0	1	3

ANALYSIS.—1. Write the subtrahend under the minuend, units of the same denomination in the same column.

2. Subtract, beginning with the lowest denomination. Since it is impossible to take 3 gi. from 2 gi., take from the minuend the 1 pt.=4 gi., and add to the 2 gi.; 2 gi.+4 gi.=6 gi.; 6 gi.−3 gi.=3 gi., which write under the column of gi.

3. Since the 1 pt. of the minuend has been taken, 0 pt. remain, and it is impossible to take the 1 pt. of the subtrahend from 0 pt. of the minuend. Therefore, take 1 qt. from the 2 qt. of the minuend, and reduce it to pt.; 1 qt.=2 pt.; 2 pt.−1 pt.=1 pt., which write under the column of pt.

4. Since 1 qt. has been taken from the 2 qt. of the minuend, 1 qt. remains. 1 qt.−1 qt.=0 qt., which write under the column of qt.

5. 18 gal.−10 gal.=8 gal.

Result, 8 gal. 0 qt. 1 pt. 3 gi.

Rule.—The pupil may easily deduce a rule from the analysis, or from an example.

8. From 10 yr. 7 mo. 8 d. subtract 5 yr. 9 mo. 20 d.

9. From 20 bu. 3 pk. 1 qt. 1 pt. subtract 15 bu. 2 pk. 6 qt. 1 pt.

10. From 1 T. take 5 cwt. 25 lb. 10 oz.

11. From 1 cd. take 50 cu. ft. 200 cu. in.

12. From 1 bbl. 5 gal. 3 qt. take 25 gal. 1 qt. 1 pt.

13. From a quadrant take a sextant.

14. From 5 lb. 8 oz. 16 pwt. take 3 lb. 9 oz. 5 pwt. 16 gr.

MULTIPLICATION.

214. *1.* Multiply 3 pk. 7 qt. 1 pt. by 9.

1st Process.

	pk.	qt.	pt.	
	3	7	1	
			9	
8 bu.	3 pk.	3 qt.	1 pt.	*Ans.*

2nd Process.

	pk.	qt.	pt.	
	3	7	1	
			9	
Products - -	27	63	9	
Quotients - -	8	4		
Amounts - -	35	67		
8 bu.	3 pk.	3 qt.	1 pt.	*Ans.*

Analysis.—1. Beginning with the lowest denomination, 9 times 1 pt.=9 pt.=4 qt. 1 pt.

2. 9 times 7 qt.=63 qt.; 63 qt.+4 qt.=67 qt.=8 pk. 3 qt.

3. 9 times 3 pk.=27 pk.; 27 pk.+8 pk.=35 pk.=8 bu. 3 pk.

Note.—In the 2nd process the work is more extended.

Rule.—Let the pupils deduce a rule, and compare it with the rule for multiplication of simple numbers.

2. Multiply 8 yd. 2 ft. 5 in. by 7.
3. Multiply 13 lb. 10 oz. Av. by 20.
4. Multiply 24° 15′ 28″ by 18.
5. Multiply 7 lb. 5 oz. 12 pwt. 8 gr. by 24.
6. Multiply 129 cu. ft. 524 cu. in. by 32.
7. Multiply 3 bales, 7 reams, 9 quires by 12.
8. Multiply 25 cu. yd. 17 cu. ft. 84 cu. in. by 40.
9. Multiply 5 T. 3 cwt. 40 lb. 8 oz. by 28.

DIVISION.

215. *1.* Divide 7 lb. 3 oz. 11 pwt. by 4.

PROCESS.

4)7 lb.	3 oz.	11 pwt.		
1 lb.	9 oz.	17 pwt.	18 gr.	

ANALYSIS. — 1. Begin with the highest denomination; one fourth of 7 lb. is 1 lb. with remainder of 3 lb.

2. 3 lb.=36 oz.; 36 oz.+3 oz.=39 oz.; one fourth of 39 oz. is 9 oz. with remainder of 3 oz.

3. 3 oz.=60 pwt.; 60 pwt.+11 pwt.=71 pwt.; one fourth of 71 pwt. is 17 pwt. with remainder of 3 pwt.

4. 3 pwt.=72 gr.; one fourth of 72 gr. is 18 gr.

Rule. — I. *Divide the number of the highest denomination given. The quotient is the first term of the result sought.*

II. *Reduce the remainder, if any, to the next lower denomination. Find the sum of the reduced remainder and the number, if any, of the same denomination of the dividend.*

III. *Divide as before, and so proceed.*

2. Divide 16 yd. 2 ft. 10 in. by 3.
3. Divide 1 T. 18 cwt. 50 lb. 8 oz. by 4.
4. Divide 60° 27′ 40″ by 12.
5. Divide 8 bu. 3 pk. 7 qt. by 6.

6. Divide 10 A. 81 sq. rd. by 9.

7. Divide 48 cu. yd. 3 cu. ft. 190 cu. in. by 36.

PROBLEMS.

1. A farmer sold 3 loads of corn at $37\frac{1}{2}¢$ per bu.; the first load contained 38 bu. 2 pk., the second 40 bu. 5 qt., the third 29 bu. 1 pk. 3 qt. What did he receive for the three loads?

2. From a cask of wine containing 40 gal., 3 gal. 1 qt. leaked out, and 20 gal. 2 qt. 1 pt. were drawn. How much remained?

3. A merchant wished to buy 400 yd. of cloth; after having purchased 5 pieces, each containing $32\frac{3}{4}$ yd., and 10 pieces each containing $18\frac{1}{4}$ yd., how many yd. must he buy?

4. If a man can walk 3 m. 150 rd. in 1 hour, how far can he walk in 7 hours?

5. What will be the size of each farm if 640 acres be divided equally into 6 farms?

6. John is 18 y. 9 mo. 10 d. old; James is one fourth as old. How much older is John than James?

7. A locomotive runs 180 miles in 8 hours. What is its average speed per hour?

8. How many books can be made from 1 bale of paper, if each sheet makes 8 leaves, and each book contains 160 pages?

9. A great gross of spools of cotton was sold for $5¢$ per spool. What was the price of the lot?

10. If 1 dose of medicine contains 2 sc. 15 gr., how many lb. Troy, etc., in 120 such doses?

11. If a man can cut 1 cd. 12 cu. ft. in 1 day, what can he cut in three eighths of a day?

12. From a bbl. of alcohol $5\frac{1}{2}$ gal. were drawn, and the remainder filled 12 bottles, each holding 3 pt. How much alcohol was there in the bbl. at first?

13. The sun moves at the rate of 15° of longitude in 1 hour. In what time does it move 100° 50′ 30″?

14. In what time does the sun move 87° 31′ 45″?

15. Through how many degrees, minutes, and seconds of longitude does the sun move in 7 h. 51 m. 47 sec.?

16. Through how many degrees, etc., does the sun move in 9 h. 59 m. 24 sec.?

17. What is the weight of a dozen silver spoons, if each weighs 16 pwt. 18 gr.?

18. Three fourths of a lb. of pure silver, mixed with 1 oz. of alloy, is made into a dozen napkin rings. What is the weight of each ring?

REVIEW QUESTIONS.

What is the reduction of a denominate number? How many kinds of reduction? Define and illustrate each. Rule for reduction descending. Rule for reduction ascending. How may each kind of reduction be proved? What is the difference between addition of simple numbers and addition of compound numbers? Give rule for addition of compound numbers. Give rule for subtraction of compound numbers. Rule for multiplication of compound numbers. Rule for division of compound numbers.

MENSURATION OF RECTANGLES AND RECTANGULAR SOLIDS.

For definitions and principles, see Arts. 183, 186, 196 and 198.

ORAL.

216. *1.* How many square inches on one side of a slate 10 in. long, 6 inches wide? How many sq. in. on both sides?

2. How many sq. in. on one page of a sheet of foolscap, 12 in. long and 8 in. wide?

3. How many sq. ft. in a blackboard 4 ft. wide and 10 ft. long?

4. How many sq. in. in a surface 8 in. by 3 in.? 9 in. by 7 in.? $7\frac{1}{2}$ in. by 4 in.? 30 in. by 8 in.?

5. How many sq. ft. in the ceiling of a room 15 ft. long and 10 ft. wide?

6. How many sq. yd. in the floor of a room 12 ft. by 11 ft.?

7. How many sq. ft. in a wall 9 ft. by 15 ft.? How many sq. yd.?

8. The cover of a book is 9 in. long, and contains 63 sq. in. How wide is it?

9. The top of a desk contains 15 sq. ft.; it is 3 feet wide. How long is it?

10. How many sq. in. in the entire surface of a brick? A brick is 8 in. by 4 in. by 2 in.

11. How many sq. in. in the entire surface of a cube 3 in. on each edge?

12. What part of a sq. yd. is a surface 2 ft. by 2 ft.?

13. What part of a sq. ft. is a surface 3 in. by 12 in.?

14. What part of a sq. yd. is a surface 1 yd. long and $\frac{1}{2}$ yd. wide?

15. What part of a sq. ft. is a surface 9 in. by 8 in.?

16. How many cu. in. in a block 5 in. long, 3 in. wide, 2 in. thick?

17. How many cu. in. in a brick?

18. How many cu. in. in a stone 10 in. by 8 in. by 4 in.?

19. How many cu. yd. in a block of marble 9 ft. long, 3 ft. wide, 2 ft. thick?

20. How many cu. ft. in a log 1 foot square and 100 ft. long?

21. How many cu. ft. in a box 3 ft. by 4 ft. by $3\frac{1}{2}$ ft.?

22. How many cu. ft. in a wall 20 ft. long, 5 ft. high, 2 ft. thick?

23. How many cu. ft. in a solid $3\frac{1}{2}$ ft. by 4 ft. by $1\frac{1}{4}$ ft.?

24. How many cu. ft. in a solid 8 ft. by $2\frac{1}{2}$ ft. by $5\frac{1}{5}$ ft.?

25. How many cu. ft. in a solid 2 yd. long, 3 ft. wide, $1\frac{1}{2}$ ft. thick?

26. A box contains 64 cu. in.; it is 4 in. wide, and 2 in. thick. How long is it?

27. A solid contains 144 cu. in.; it is 2 in. thick, 6 in. wide. How long is it?

WRITTEN.

28. How many sq. ft. in a surface 172 ft. long, 80½ ft. wide?

29. How many sq. yd. in a surface 324 ft. long, 81 ft. wide?

30. How many sq. rd. in a surface 320 rd. by 1280 rd.? How many acres? How many square miles?

31. How many sq. ft. in the walls of a room 21 ft. long, 12 ft. wide, 9 ft. high?

Find the product of the *perimeter* of the room, or the entire length of its four walls, and its height; thus: (21 ft.×2)+(12 ft.×2) =66 ft., the *perimeter* of the room; 66×9= the entire surface of the four walls.

32. How many sq. ft. in the four walls of a room 24 ft. long, 18 ft. wide, 12 ft. high? How many sq. yd.?

33. Find the cost of plastering the walls and ceiling of the room described in Ex. 31, at 62½¢ per sq. yd.

34. Find the cost of plastering the walls and ceiling of the room described in Ex. 32, at 37½¢ per sq. yd.

35. How many sq. yd. in floor, walls, and ceiling of a room 48 ft. long, 36 ft. wide, 15 ft. high?

36. Cost of laying a sidewalk 200 ft. long, 14 ft. wide, at 8¢ per sq. ft.?

37. How many sq. yd. in a floor 18 ft. long, 12 ft. wide? How many sq. yd. of carpeting necessary to cover the floor?

38. How many yd. of carpeting, 1 yd. wide, necessary to cover a floor 12 ft. long and 9 ft. wide?

39. How many yd. of carpeting, ½ yd. wide, necessary to cover the floor of a room 12 ft. long and 9 ft. wide?

40. How many yd. of carpeting, ¼ yd. wide, necessary to cover the floor of a room 12 ft. long and 9 ft. wide? How many yd. ¾ yd. wide?

41. Cost of carpeting, ¾ yd. wide, necessary to cover a floor 21 ft. long by 12 ft. wide, at $1.50 per yd.?

Carpets are sold at a certain price per yd. *in length.* A piece of

carpet $\frac{3}{4}$ yd. wide, 20 yd. long, contains 15 sq. yd.: but the cost at \$2 per yd. is $20 \times \$2$, not $15 \times \$2$.

42. Cost, at \$2.25 per yd., of carpet $\frac{5}{8}$ yd. wide, necessary to cover a floor 40 ft. long, 24 ft. wide?

43. A blackboard containing 96 sq. ft. is 4 ft. wide. How long is it?

44. A street 80 ft. wide contains 1 acre. How many ft. long is it?

45. How many squares, each 6 in. on a side, can be cut from a sheet of paper 4 ft. square?

46. Cost, at \$2.50 per acre, of a tract of land containing 6 sq. miles?

47. How many cu. ft. in a wall 24 ft. long, 10 ft. high, 2 ft. thick?

48. How many cu. ft. in a wall 96 ft. long, $18\frac{3}{4}$ ft. high, $2\frac{1}{2}$ ft. thick?

49. How many cu. in. in a block 3 ft. by 5 ft. by $1\frac{2}{3}$ ft.?

50. How many cu. ft. in a stone 144 in. long, 24 in. wide, 8 in. thick?

51. How many cubes, each 3 in. on each edge, can be cut from 1 cu. ft.?

52. How many cu. ft. in a pile of wood 16 ft. long, 4 ft. wide, 4 ft. high?

53. How many cords in a pile of wood 24 ft. long, 4 ft. wide, 4 ft. high?

54. What is the value, at \$6.75 per cord, of a pile of wood 32 ft. long, 8 ft. wide, 4 ft. high?

55. What is the cost of sawing, at $87\frac{1}{2}¢$ per cord, a pile of wood 12 ft. long, 4 ft. wide, 6 ft. high?

56. How many perch of stone in a pile 99 ft. long, 24 ft. wide, 10 ft. high?

57. Find value of stone described in Ex. 56, at $\$3.12\frac{1}{2}$ per perch?

58. How many perch of masonry in a wall $49\frac{1}{2}$ ft. long, 8 ft. high, 4 ft. thick?

59. Cost of building a wall of stone $24\frac{3}{4}$ ft. long, 7 ft. high, 2 ft. thick, at \$4.62½ per perch?

60. How many bricks in a wall 24 ft. long, 8 ft. high, 3 ft. thick?

Use cancellation, thus: $\frac{24\times8\times3\times1728}{.2\times4\times8}=$.

61. How many bricks in a pile 40 ft. by 20 ft. by 12 ft.?

62. Find the cost of brick necessary to build a wall 80 ft. long, 60 ft. high, $2\frac{1}{2}$ ft. thick, at \$8 per M.?

63. Cost, at \$9.50 per M., of the bricks necessary to build the walls of a house in the form of a rectangle, 40 ft. long, 32 ft. high, 24 ft. wide, the walls 2 ft. thick?

NOTE.—Take the outside measurement, which is given. Make no allowance for doors or windows. This will give the *mason's* measurement, which is rather greater than the actual measurement. There are 21 bricks in a cubic foot, allowing for mortar.

64. How many cu. in. in 40 gallons?

65. How many cu. in. in 5 bbl.?

66. How many cu. in. in 1 hhd.?

67. How many cu. in. in 5 bushels?

68. How many cu. ft. in 20 bushels?

69. How many gal. in a box 40 in. by 24 in. by 18 in.?

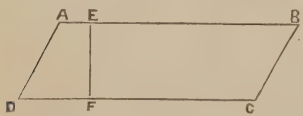
70. How many gal. in a box 3 ft. by 2 ft. by $1\frac{1}{2}$ ft.?

71. How many gal. in a tank 8 ft. by 6 ft. by 4 ft.

72. How many bushels may be contained in a bin 20 ft. by 8 ft. by 12 ft.?

MENSURATION OF PARALLELOGRAMS.

217. A ***Parallelogram*** is a plane four-sided figure, whose opposite sides are parallel and equal.



A B C D is an oblique angled parallelogram.

Squares and rectangles are varieties of the parallelogram.

218. The ***Base*** of any plane figure is the side upon which it is supposed to rest. Parallelograms are

considered as having two bases, called *upper* and lower *bases.*

A B and C D are the bases of the parallelogram A B C D.

219. The ***Altitude*** of a parallelogram is the perpendicular distance between its bases.

E F is the altitude of the parallelogram A B C D.

In rectangles, either end or side is the altitude, because the end or side is perpendicular to the base.

220. *The area of a parallelogram is equal to the product of its base and altitude.*

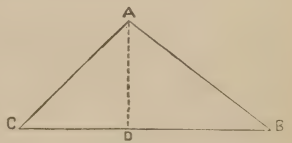
1. Find the area of a parallelogram having a base of 12 ft., altitude of 8 ft.

2. Find area of parallelogram having base 50 ft., altitude 25 ft.

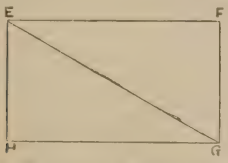
MENSURATION OF TRIANGLES.

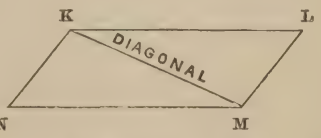
221. A ***Triangle*** is a plane figure bounded by three straight lines.

A B C is a triangle. C B is its base; the angle A, opposite the base, is its *vertex;* A D, the perpendicular distance from the vertex to the base, is its altitude.



222. The ***Diagonal*** of a parallelogram is the straight line joining two opposite angles.

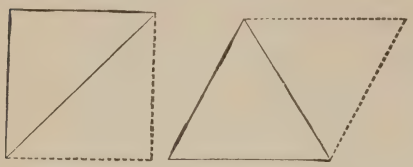




E G is the diagonal of the rectangle E F G H. K M is the diagonal of the parallelogram K L M N.

223. *The diagonal of a parallelogram divides the parallelogram into two equal triangles.*

The triangles E F G and E G H are equal. The triangles K L M and K M N are equal.



The pupil may draw any triangle, and by annexing to it, in the manner shown, an equal triangle, may form a parallelogram.

224. Since the area of a parallelogram is equal to the product of its base and altitude; and since every parallelogram may be divided into two equal triangles having the same base and altitude as the parallelogram, it follows that

The area of a triangle is one half of the product of its base and altitude.

Find the area of the following triangles:

1. Base 6; altitude 4.
2. Base 12; altitude 8.
3. Base 40; altitude 15.
4. Base 60; altitude 30.
5. Base 85; altitude 20.

6. The base of a triangle is $12\frac{1}{2}$; its altitude is 25. What is its area?

7. The area of a triangle is 625; its altitude is 50. What is its base?

8. The area of a triangle is 360; its base is 60. What is its altitude?

9. The area of a parallelogram is 360; its base is 60. What is its altitude?

REVIEW QUESTIONS.

Define parallelogram; triangle. Define base of parallelogram; base of triangle. Define altitude of parallelogram. Define diagonal. How may the area of a parallelogram be found? Of a triangle? If a parallelogram and a triangle have same base and altitude, how do their areas compare?

SECTION XII.

PERCENTAGE.

Art. 225. Per Cent means *hundredths.*

1 per cent is $\frac{1}{100}$; 6 per cent is $\frac{6}{100}$, etc.

226. Rate per Cent means a certain number of hundredths.

In the expression 6 per cent, 6 is the *rate*. The *rate* is the numerator of a fraction of which the denominator is always 100. Instead of the words *per cent* the sign % is frequently used.

227. Rate per cent may be written as a common fraction, as a decimal fraction, or as per cent.

Thus:

$\frac{1}{100}$=.01=1%; the last, 1%, is read, *one per cent.*

$\frac{6}{100}$=.06=6%.

$\frac{7\frac{1}{2}}{100}$=.07$\frac{1}{2}$ or .075=7$\frac{1}{2}$%; the last is read *seven and one half per cent.*

$\frac{12\frac{1}{2}}{100}$=.12$\frac{1}{2}$ or .125=12$\frac{1}{2}$%.

$\frac{125}{100}$=1.25=125%.

$\frac{\frac{1}{2}}{100}$=.005=$\frac{1}{2}$%, the last is read *one half of one per cent.*

Observe that the decimal point is not used when the words *per cent* are used, or when the sign % is used, unless a *fractional* per cent is meant. Thus, 5% equals .05, not .5%. .5% means 5 tenths of one per cent, and is otherwise written $\frac{1}{2}$%, or .00$\frac{1}{2}$.

Express as per cent:

1. $\frac{8}{100}$.	*5.* $\frac{50}{100}$.	*9.* .06.	*13.* 1.18.
2. $\frac{6}{100}$.	*6.* $\frac{150}{100}$.	*10.* .10.	*14.* 2.60.
3. $\frac{20}{100}$.	*7.* $\frac{225}{100}$.	*11.* .08$\frac{1}{3}$.	*15.* .005.
4. $\frac{16}{100}$.	*8.* $\frac{7\frac{1}{2}}{100}$.	*12.* .12$\frac{1}{2}$.	*16.* .00375.

Express decimally:

1. 10%.	*6.* $8\frac{1}{2}$%.	*11.* 100%.	*16.* $7\frac{1}{5}$%.
2. 15%.	*7.* $6\frac{1}{4}$%.	*12.* 125%.	*17.* 224%.
3. 5%.	*8.* $5\frac{3}{8}$%.	*13.* $112\frac{1}{2}$%.	*18.* $162\frac{1}{2}$%.
4. 6%.	*9.* $12\frac{1}{2}$%.	*14.* 300%.	*19.* $137\frac{1}{2}$%.
5. 25%.	*10.* 40%.	*15.* 375%.	*20.* $106\frac{1}{4}$%.

228. The ***Base*** is the number of which the per cent is computed.

229. The ***Percentage*** is that per cent of the Base which is indicated by the *Rate.*

1. A man having 500 bushels of wheat, sold 8% of it. How many bushels did he sell?

PROCESS.
$500 \times .08 = 40.$

SOLUTION.—He sold .08 of 500 bushels, which is 40 bushels.

In this example 500 bushels is the *Base;* 8% is the *Rate per cent;* 40 bushels is the *Percentage.*

Observe that this is but a simple problem in multiplication of decimals.

230. The *Base* is the *Multiplicand.*

The *Rate per cent* is the *Multiplier.*

The *Percentage* is the *Product.*

231. As in simple multiplication, if any two of these terms are given, the other may be found, since the Base and Rate are *factors* of the Percentage.

2. A man having 500 bushels wheat sold 40 bushels. What per cent. of his wheat did he sell?

PROCESS.
$40 \div 500 = .08.$

SOLUTION.—An inspection of example 1 will show that 40 bushels is the *product* of two factors, the *Base* and the *Rate.* Dividing the product, 40, by the given factor, 500, the quotient is the *Rate,* 8%.

3. A man sold 40 bushels of wheat, which was 8% of what wheat he had. How many bushels had he?

PROCESS.
$40 \div .08 = 500.$

SOLUTION.—Comparing this example with example 1 and 2, it will be seen that the factor required here is the Base. Dividing the product, 40, by the given factor, .08, the quotient is the *Base,* 500.

CASE I.—Base and Rate per cent. given, to find the Percentage.

232. Rule.—*Find the product of the Base and the Rate expressed as a decimal.*

Study example 1.

CASE II.—Base and Percentage given, to find the Rate.

233. Rule.—*Find the quotient of the Percentage divided by the Base.*

Study example 2.

CASE III.—Percentage and Rate given to find the Base.

234. Rule.—*Find the quotient of the Percentage divided by the Rate.*

Study example 3.

ORAL.

4. What is 10% of 50? Of 60? Of 80? Of 100?

5. What is 20% of 40? Of 80? Of 75? Of 60?

6. Coffee was bought for 40¢ and sold at a gain of 20%. What was the gain?

7. A knife that cost $1 was sold at an advance of 50%. What was the gain?

8. A book costing $.60 was sold at a loss of 30%. What was the loss?

9. A farmer having 200 sheep lost 40% of them. How many did he lose?

10. John had 80 marbles, and sold 25% of them to James. How many had he remaining?

11. What per cent is 8 of 16? 16 of 8?

12. What per cent is 5 of 25? 25 of 5?

13. What per cent is 80 of 20? 20 of 80?

14. 15 is what per cent of 60? 60 of 15?

15. Mary having 20 peaches, gave 10 to her playmates. What per cent of her peaches did she give to them.

16. A regiment of 1000 men lost 50 men in battle. What per cent of its number did it lose?

17. I sold my gold pen, which cost \$2, for \$1.50. What per cent did I lose?

18. Julia's watch is worth \$50; Annie's is worth \$75. What per cent of the value of Annie's watch is the value of Julia's?

19. What per cent of the value of Julia's watch is the value of Annie's?

20. John bought a sled for \$2, and lost it. What per cent of his investment did he lose?

21. Mr. Roberts invested \$2000 in business, and gained 10% of his original investment every year. How many dollars did he gain in 1 year? In 5 years?

22. Mr. Tracy gained every year 20% of an investment. In how many years will he gain 40%? In how many years will he double his capital?

23. Percentage 20; rate per cent 10. Find base.

24. Percentage 30; rate per cent 25. Find base.

25. Percentage 60; rate per cent 6. Find base.

26. Percentage 56; rate per cent 8. Find base.

27. \$20 is 20% of John's money. How much has he?

28. \$50 is 12½% of the value of a horse. What is his value?

29. A book was sold at a gain of \$1.50, which was 30% of its cost. What was its cost?

30. A man traveled 40 miles by stage, which was 20% of his whole journey. How many miles in the journey?

235. When the rate per cent is an aliquot part of 100, the rules given above need not be followed.

31. John had \$50, and lost 10% of it. How many dollars did he lose?

SOLUTION.— $10\% = \frac{10}{100} = \frac{1}{10}$. John lost $\frac{1}{10}$ of his money, that is, \$5.

32. James having 24 books, sold 37½% of them. How many did he sell?

SOLUTION.— $37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{3}{8}$. James sold $\frac{3}{8}$ of his books, that is, 9 books.

33. Mary is 15 years old; Jane is 10 years old. What per cent of Mary's age is Jane's age?

SOLUTION.—Jane's age is $\frac{10}{15}$ or $\frac{2}{3}$ of Mary's age. $\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3}\%$.

34. Henry gave his sister 3 apples, which were 25% of what he had. How many had he?

SOLUTION.— $25\% = \frac{25}{100} = \frac{1}{4}$. 3 was $\frac{1}{4}$ of Henry's apples whole number was 4 times 3 apples, or 12 apples.

TABLE.

$1\% = \frac{1}{100}$.	$8\frac{1}{3}\% = \frac{8\frac{1}{3}}{100} = \frac{1}{12}$.
$2\% = \frac{2}{100} = \frac{1}{50}$.	$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$.
$4\% = \frac{4}{100} = \frac{1}{25}$.	$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{3}{8}$.
$5\% = \frac{5}{100} = \frac{1}{20}$.	$62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{5}{8}$.
$10\% = \frac{10}{100} = \frac{1}{10}$.	$87\frac{1}{2}\% = \frac{87\frac{1}{2}}{100} = \frac{7}{8}$.
$20\% = \frac{20}{100} = \frac{1}{5}$.	$16\frac{2}{3}\% = \frac{16\frac{2}{3}}{100} = \frac{1}{6}$.
$25\% = \frac{25}{100} = \frac{1}{4}$.	$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{1}{3}$.
$50\% = \frac{50}{100} = \frac{1}{2}$.	$66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100} = \frac{2}{3}$.
$75\% = \frac{75}{100} = \frac{3}{4}$.	

NOTE.—The pupil should be made perfectly familiar with the above table.

35. Find 25% of 80; of 100; of 400; of 50.

36. Find $33\frac{1}{3}\%$ of 60; of 45; of 300; of 75.

37. Find $12\frac{1}{2}\%$ of 40; of 160; of 64; of 96.

38. Find $37\frac{1}{2}\%$ of 80; of 320; of 400; of 640.

39. What per cent of 60 is 40? Of 80 is 60?

40. 5 is 20% of what number?

41. 12 is $66\frac{2}{3}\%$ of what number?

42. 14 is $87\frac{1}{2}\%$ of what number?

43. 15 is $16\frac{2}{3}\%$ of what number?

WRITTEN.

Find

44. 16% of 200.

45. 13% of 400.

46. 27% of 395.

47. 38% of 750.

48. 5% of 125.

49. 9% of 840.

50. 125% of 450.

51. 230% of 620.

52. A man sowed in oats 30% of his farm of 160 acres. How many acres did he sow in oats?

53. Mr. Roberts is 80 years old; he lived 40% of his life in England. How many years did he live in England?

54. 24% of a flock of sheep numbering 2800 was sold. How many sheep were sold?

55. A man bought a horse for $200, and sold him so as to gain 10% of his cost. What did he gain?

NOTE.—The COST is always the BASE. The GAIN OR LOSS is the PERCENTAGE.

56. A house was bought for $4000. It is now valued at 125% of its cost. What is its value?

57. A lawyer receives 8% of all money that he collects. What is his fee on a collection of $500?

58. A merchant insures his goods for $5000, paying $\frac{3}{4}\%$. What does he pay?

SUGGESTION.—First find 1% of $5000, by dividing by .01; then take $\frac{3}{4}$ of this quotient.

NOTE.—The money paid to insure property is called *premium*.

59. What is the premium paid for insuring a house worth $8000, for one half its value, at $1\frac{1}{2}\%$?

60. A merchant insured his store for $2000, at $1\frac{3}{4}\%$, and his stock of goods for $50000, at $\frac{7}{8}\%$. Find his total premium.

61. A ship was bought for $18000; repaired at an expense of $4000, and sold at a gain of 9% on total cost. What was the gain?

Find the rate:

62. Base 400; percentage 32.

63. Base 900; percentage 630.

64. Base 846; percentage 282.

65. Base 725; percentage 125.

66. John having $60, gave his brother $12. What per cent of his money did he give to his brother?

67. A house valued at $2000 was damaged to the extent of $800. What was the per cent of the damage?

68. A watch was bought for $80, and sold for $100. What was the gain per cent?

69. A grocer buys sugar at 8 cents, and sells it at 9 cents. What is his gain %?

70. When a man sells for 10 cents what has cost him 12 cents, what is his loss %?

71. A vessel which cost $40000, was sold for $35000. What was the loss %?

72. A collector charged $72 for collecting $900. What was his charge %?

73. A house was insured for $8500. The premium was $127.50. Find the rate.

Find the base:

74. Percentage 42; rate 6%.

75. Percentage 75; rate 5%.

76. Percentage 91; rate 7%.

77. Percentage $87\frac{1}{2}$; rate $12\frac{1}{2}\%$.

78. A man lost $50 which was 10% of his money. How much had he at first?

79. Mary is 8 years old; her age is 20% of her mother's age. How old is her mother?

80. A merchant sold $12000 worth of goods in December, which was 15% of what he sold during the entire year. What were his annual sales?

81. The gain on the sale of a house was $200, which was $12\frac{1}{2}\%$ of its cost. What was its cost?

82. The premium for insuring a house was $60; the rate was $1\frac{1}{2}\%$. For what was the house insured?

83. The taxes on a certain lot are $30 per year; the levy is 15 mills on the dollar. What is the assessed value of the lot?

84. The loss on the sale of a horse was $50, which was 40% of his cost. What was his cost?

REVIEW QUESTIONS.

What is meant by per cent? By rate per cent? In what way may per cent be expressed? Explain the use and non-use of the decimal point in writing rate per cent? Define base. Define percentage. To what term in multiplication does each term in percentage correspond? How may each term be found? Which terms in percentage are factors? State and explain Case I. Case II. Case III. To which term in percentage does the *cost* of an article correspond? The *gain* or *loss?* Define premium.

INTEREST.

236. ***Interest*** is the compensation or payment due one party (the lender) from another party (the borrower) for the use of money.

237. The ***Principal*** is the sum of money for whose use a compensation is made.

238. The ***Rate of Interest*** is a certain rate per cent, which the interest is of the principal, for a specified time, usually one year.

239. The ***Amount*** is the sum of the principal and interest.

ORAL.

1. What is the interest of $100 for 1 year, at 6%?

SOLUTION.—The interest is 6% of $100, that is, $6.

2. What is the interest of $40 for 3 years, at 10%?

SOLUTION I.—The interest of $40 for one year, at 10% interest, is $4; for 3 years it is 3 times $4, that is, $12.

SOLUTION II.—The interest of $1 at 10% for one year is 10 cents; for three years it is 30 cents. The interest on $40 for 3 years, at 10%, is 40 times 30 cents, that is, $12.

What is the interest of

3. $80 for 1 year at 8%?
4. $50 for 2 years at 4%?
5. $200 for 4 years at 10%?
6. $900 for 3 years at 7%?
7. $300 for 5 years at 9%?
8. $75 for 4 years at 5%?
9. $10 for 4 years at 6%?
10. $3.50 for 2 years at 10%?
11. $7.50 for 4 years at 5%?
12. $12.50 for 2 years at 8%?
13. 3.33\frac{1}{3}$ for 3 years at 10%?
14. $50 for 2 years at 7$\frac{1}{2}$%?
15. $30 for 3 years at 5$\frac{1}{3}$%?
16. $1 for 10 years at 10%?
17. $3 for 8 years at 12$\frac{1}{2}$%?
18. $5 for 4 years at 6$\frac{1}{4}$%?

19. What is the interest of $30 for 1 year 4 months at 9%?

SOLUTION.—The interest of $30 for 1 year at 9% is $2.70; for 4 mos. ($\frac{1}{3}$ of a year) the interest is $\frac{1}{3}$ of $2.70, or 90 cents.

$2.70+.90=$3.60.

20. What is the interest of $60 for 1 yr. 3 mos. at 6%?
21. What is the interest of $400 for 2 yrs. 6 mos. at 10%?
22. What is the interest of $900 for 3 yrs. 2 mos. at 6%?
23. What is the interest of $150 for 1 yr. 9 mos. at 6%?
24. What is the interest of $80 for 4 yrs. 4 mos. at 8%?
25. What is the interest of $600 for 5 mos. at 8%?

SUGGESTION.—Find interest for 1 year, then for 1 month, then for 5 months.

26. Find interest of $120 for 7 mos. at 4%.
27. Find interest of $360 for 11 mos. at 9%.

WRITTEN.

28. Find interest of $240 for 3 years 7 months at 8%.

PROCESS.

12	$240
	.08
	43
	$68.80

SOLUTION I.—Interest for 1 year=$240×.08=$19.20. Interest for $3\frac{7}{12}$ yrs.=$19.20×$3\frac{7}{12}$=$68.80.

SOLUTION II.—Express, in form for cancellation, the interest for 1 year, $240×.08.

Express the division of this product by 12, which is the interest for 1 month. Express the multiplication of this quotient by 43, the number of months (3 yrs.+7 mos.=43 mos.).

By cancellation, the result is $68.80.

29. Find interest of $360 for 2 yrs. 5 mos. at 6%.
30. Find interest of $600 for 3 yrs. 11 mos. at 8%.
31. Find interest of $720 for 1 yr. 9 mos. at 8%.
32. Find interest of $480 for 3 yrs. 6 mos. at 10%.
33. Find interest of $90 for 4 yrs. 5 mos. at 6%.
34. Find interest of $72 for 5 yrs. 3 mos. at 8%.
35. Find interest of $418 for 18 days at 6%.

PROCESS.

12	$480
30	.06
	18
	$1.44

SOLUTION.—Express, in form for cancellation, the interest for 1 year, $480×.06.

Express the division of this product by 12, which result is the interest for 1 month.

Express the division of this quotient by 30, which result is the interest for 1 day.

Express the multiplication of this quotient by 18, which result is the interest for 18 days.

By cancellation, the result is $1.44.

36. Find interest of $300 for 20 days at 8%.
37. Find interest of $450 for 19 days at 10%.
38. Find interest of $1200 for 16 days at 7%.
39. Find interest of $1500 for 25 days at 9%.
40. Find interest of $840 for 7 months 21 days at 6%.

PROCESS.

12	$840
	.06
	7.7
	$32.34

SOLUTION.—Consider the 21 days as .7 of a month. Proceed as in example 28.

41. Find interest of $660 for 3 mos. 18 days at 6%.
42. Find interest of $340 for 5 mos. 24 days at 8%.
43. Find interest of $800 for 9 mos. 15 days at 4%.
44. Find interest of $140 for 7 mos. 12 days at 5%.
45. Find interest of $600 for 11 mos. 21 days at 7%.
46. Find interest of $500 for 6 mos. 27 days at 10%.
47. Find interest of $600 for 5 yrs. 3 mos. 11 days, at 5%.

PROCESS.

12	$600.	
30	.05	
	101	
3	25.25	
	$8.41⅔	

SOLUTION.—Interest on $600 for 5 yrs.=$600×.05×5=$150. 3 mos+11 days=101 days. Find interest for 101 days by cancellation, as in example 35, $8.41⅔.

$150+$8.41⅔=$158.41⅔.

Find interest of

48. $960 for 2 yrs. 8 mos. 7 days, at 10%.

49. $1440 for 3 yrs. 5 mos. 15 days, at 9%.

50. $60 for 2 yrs. 3 mos. 20 days, at 7½%.

SUGGESTION.— 7½%=$\frac{15}{2}$%.

51. $1200 for 4 yrs. 6 mos. 5 days, at 4½%.

52. $195 for 8 yrs. 1 mo. 3 days, at 6¼%.

53. $180.75 for 1 yr. 2 mos. 10 days, at 6%.

54. $47.60 for 2 yrs. 3 mos. 18 days, at 7%.

55. Find the amount of $325.50, for 4 yrs. 6 mos. 15 days, at 6%.

56. Find the amount of $480.90 for 3 yrs. 9 mos. 10 days, at 8%.

57. I bought a lot for $800, and after 2 yrs. 6 mos. I sold it for $1200. What was the gain, if money was worth 10%?

58. What amount of money will pay a note of $500, due in 3 yrs. 6 mos. 20 days, at 10%?

59. I loaned Mr. Jones $800, which he returned in 8 mos. 10 days, with 8% interest. What did he pay me?

60. What is the annual income from an investment of $10000, which pays 8½%?

REVIEW QUESTIONS.

Define interest. Define principal. Define rate of interest. Define amount. What part of the principal is the interest for one year?

MISCELLANEOUS EXERCISES IN PERCENTAGE.

A. Find commission, or premium, or gain or loss, on numbers in first column, at 1%, $\frac{1}{4}$%, $\frac{1}{2}$%, $\frac{3}{4}$%, $1\frac{1}{2}$%, $2\frac{2}{3}$%, $6\frac{1}{4}$%, $8\frac{1}{3}$%.

B. Find interest on each of the following principals for the time indicated, at 4%, $4\frac{1}{2}$%, 5%, 6%, 7%, $7\frac{1}{2}$%, 8%, 9%, 10%.

1.—$458.75	1y.	6m.	15d.	16.—$428.40	1y.	9m.	27d.
2.—$680.50	2y.	9m.	18d.	17.—$600.00	2y.	5m.	6d.
3.—$320.82	3y.	2m.	9d.	18.—$325.15	3y.	7m.	
4.—$500.00	1y.	9m.	21d.	19.—$216.80	2y.	3m.	3d.
5.—$753.375		8m.	20d.	20.—$824.60		11m.	15d.
6.—$873.40	5y.	2m.	3d.	21.—$324.20	4y.	6m.	18d.
7.—$157.25	3y.	5m.	12d.	22.—$ 80.50	1y.	1m.	5d.
8.—$ 32.30	1y.	4m.	27d.	23.—$192.60			29d.
9.—$750.00	3y.	8m.		24.—$455.24	2y.	0m.	17d.
10.—$324.125	2y.	5m.	6d.	25.—$128.125	1y.	3m.	22d.
11.—$862.17	2y.	9m.	17d.	26.—$840.75	3y.	5m.	13d.
12.—$920.09	1y.	10m.	3d.	27.—$224.375	2y.	9m.	21d.
13.—$382.11	3y.	6m.	19d.	28.—$800.00	7y.	7m.	7d.
14.—$809.05	2y.	7m.	21d.	29.—$925.06	10y.	6m.	
15.—$395.60	1y.	10m.	12d.	30.—$708.20	2y.	8m.	24d

C. Find numbers of which the numbers below are 1%, $1\frac{1}{2}$%, 2%, $2\frac{1}{4}$%, $2\frac{1}{2}$%, 3%, 4%, 5%, $12\frac{1}{2}$%, $16\frac{2}{3}$%.

D. Find what rate per cent each of the following numbers is of the succeeding number.

1.— 40	16.—525	31.—711
2.— 60	17.—620	32.—425
3.— 64	18.—720	33.—632
4.— 84	19.—382	34.—375
5.— 96	20.—160	35.—900
6.—132	21.—501	36.—432
7.—240	22.—921	37.—723
8.—276	23.—308	38.—522
9.—300	24.—427	39.—735
10.—312	25.—390	40.—640
11.—342	26.—570	41.—832
12.—426	27.—891	42.—729
13.—405	28.—927	43.—324
14.—414	29.—800	44.—108
15.—510	30.—639	45.—216

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